Preprints of the 4th IFAC Workshop on Distributed Estimation
and Control in Networked Systems
September 25-26, 2013. Rhine-Moselle-Hall, Koblenz, Germany

Dissipativity-Based Teleoperation with Time-Varying Communication Delays ⋆

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Abstract:
We analyze the effects of communication delays in teleoperation systems using dissipativity theory along with explicit models of the operator and robot. We utilize a simple model of the operator's behavior that describes human pointing motions, as generated by an interface such as a mouse pointer or tablet, and we use a robot model that is suitable for mobile robots or robotic manipulators. Using dissipativity conditions for stability, we show that the communication delays can be compensated for in the robot controller with a relatively simple extension to a controller designed for the situation without delays. We also show that the communication delays can lead to problems for human pointing in certain situations; specifically, if the operator overshoots their target, it may lead to instability unless corrective action is taken by the user interface. Simulation is shown to validate the results.

Keywords: Teleoperation, Time delay, Human-machine interface, Robotics, Dissipativity.

1. INTRODUCTION

Teleoperated robots often have communication delays to and from and their operators as the result of signal propagation times and bandwidth constraints. These delays can seriously degrade control performance and even lead to instability, particularly when the delays are large relative to the speed of the dynamics of the robot. While it may not be possible to eliminate these communication delays, we can reduce their effects by compensating for them in the design of the robot controller and the user interface.

Our goal is to design the teleoperation system shown in Fig. 1 to be stable despite the communication delays. In this setup, the human operator interacts with the user interface (UI) which transmits control information over the communication link to the controller on-board the robot, which in turn transmits back feedback information to the operator through the user interface. The communication link delays the messages by an unknown length of time in either direction. We specifically consider the case where the control objective is for the robot to track a trajectory specified by the operator in real time; in other words, the operator controls the robot's pose.

We use dissipativity theory along with explicit models of the operator and robot to analyze the effects of the communication delays. We describe the operator's behavior with a simple dynamics model for human pointing motions that is applicable to a variety of pointing interfaces, such as a mouse cursor or a tablet. We adopt a robot dynamics model that is suitable for mobile robots or robotic manipulators. Using dissipativity conditions for stability, we show that the communication delays can be compensated for in the robot controller with a relatively simple extension to a controller designed for the situation without delays. We also show that the communication delays can lead to problems for human pointing in certain situations; specifically, if the operator overshoots their target, it may lead to instability unless corrective action is taken by the UI.

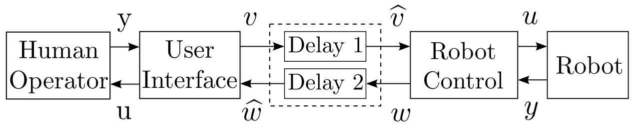

Fig. 1. Block diagram of teleoperation system with communication delays.

There has been a great deal of work in analyzing the effects of delays and designing controllers to compensate for them. Our dissipativity-based approach is most closely related with the work on the scattering transform and its associated passivity conditions for stability of delayed systems. The scattering transform, originally presented in Anderson and Spong (1989) and subsequently reformulated in Niemeyer and Slotine (1991), has been used extensively to deal with unknown communication delays in haptic bilateral teleoperators, see Chopra and Spong (2004), and has also been extended to the cases of time-varying delays in Niemeyer and Slotine (1998); Lozano et al. (2002); Kottenstette and Antsaklis (2007) and discrete time delays in Berestesky et al. (2004); Chopra et al. (2008). Our work is also closely related to finite gain L_2 approaches, as in Chopra and Spong (2007), and reproduces some of the results in that area.

⋆ The research work is supported by ONR grants N00014-09-1-1074 and N00014-10-10712 (YIP), and NSF grants ECCS- 0841195 (CAREER), CNS-0931576, and OCE-1032285.

This work has three novel contributions. First, we incorporate a model for human pointing motion into the classical delayed teleoperation setup. While we are not the first to use assumptions about human behavior to analyze teleoperation systems, to the best of our knowledge, this work is the first to utilize a dynamics model for human pointing behaviors. Second, using dissipativity-based methods, we are able to show under what conditions the teleoperation system may become unstable when no delay compensation is applied. These dissipativity methods reframe and extend the existing work for passivity-based teleoperation. Third, we design delay compensation for the robot controller and show preliminary results for the UI design.

The remainder of the paper is organized as follows. In Section 2 we present some background information relevant to our work. In Section 3 we show that communication delays are dissipative systems with respect to a particular supply rate, and we use this supply rate to derive new stability conditions for systems with communication delays. In Sections 4 and 5 we analyze and design the UI and robot control, respectively, using the derived stability conditions, and in Section 6 we present a summary of this analysis. In 7 we present the results of simulation of the teleoperation system. In Section 8 we finish with conclusions of this work.

2. BACKGROUND

Here we present some background information relevant to our approach. We show a standard model for robotic vehicles and detail the derivation of a commonly used trajectory controller based on this model without consideration of communication delays. We also give a short introduction to dissipative systems, the basis for our approach to delay compensation.

2.1 Robotic Vehicle Dynamics and Control

In order to design the robot controller, we must start with a reasonable model for the system. We use a general purpose rigid body dynamics model that can be used for robotic manipulators Slotine (1988) and many types of mobile robots Murray and Li (1994); Fossen (1994); Fjellstad (1994)

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q,\dot{q})\dot{q} + G(q) = \tau \qquad (1)$$

where $q, \dot{q}, \tau \in \mathbb{R}^n$ are the generalized coordinates, velocity, and forces of the vehicle, respectively. For simple vehicles, q is the position and orientation of the vehicle, and for manipulators, q is the joint angles.

We make a few assumptions about this model that are often true in general From et al. (2010):

$$\dot{M}(q) - 2C(q,\dot{q}) \text{ is skew symmetric} \qquad (2)$$

$$M(q) \text{ and } D(q,\dot{q}) \text{ are symmetric positive definite} \qquad (3)$$

We also mention that there is some freedom in how $C(q,\dot{q})$ can be represented, and (2) only applies to particular representations.

For this model, the controller shown below guarantees tracking of a desired trajectory for robotic vehicles under normal circumstances, but will not necessarily achieve this when there are communication delays in the feedback path. For simplicity, the version presented here assumes that the system dynamics M, C, D, and G are known; however, the adaptive version of this controller is nearly identical for our purposes; see Fossen (1994) for a derivation of the adaptive version of this controller. Connecting the definitions above to the setup in Fig. 1, we define

$$y = w = x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \qquad \widehat{v} = v = x_d = \begin{bmatrix} q_d \\ \dot{q}_d \end{bmatrix} \qquad u = \tau$$

which corresponds to state feedback, reference input of the the desired state of the vehicle without any delays, and direct application of control forces to the vehicle.

We define some auxiliary variables that simplify the notation. The error

$$e(t) = [\dot{q}(t) - \dot{q}_d(t)] + \lambda[q(t) - q_d(t)]$$

where λ is some positive constant, referred to as the bandwidth. Note that, $e(t) \to 0$ implies that $q(t) \to q_d(t)$ and $\dot{q}(t) \to \dot{q}_d(t)$. With this, our tracking goal is simply to make e asymptotically stable to the origin.

The virtual reference trajectory q_r is defined by

$$\dot{q}_r(t) = \dot{q}(t) - e(t)$$

and will appear in the derivation of the controller associated with feedforward terms.

With these definitions, we will now analyze the error dynamics while at the same time trying to select u with the intent of making e asymptotically stable. Choosing

$$V(e) = \frac{1}{2} e^\top M e$$

Taking the derivative of this with respect to time

$$\dot{V}(e) = e^\top M \dot{e} + \frac{1}{2} e^\top \dot{M} e$$

since by assumption M is symmetric. We can use the fact that $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric to show

$$\begin{aligned} \frac{1}{2} e^\top \dot{M}(q) e &= \frac{1}{2} e^\top \left(\dot{M}(q) - 2C(q,\dot{q}) \right) e + e^\top C(q,\dot{q}) e \\ &= e^\top C(q,\dot{q}) e \end{aligned}$$

Replacing terms in the previous expression, we have

$$\dot{V}(e) = e^\top M(q)(\ddot{q} - \ddot{q}_r) + e^\top C(q,\dot{q}) e$$

Substituting in the dynamics for $M(q)\ddot{q}$ and regrouping

$$\begin{aligned} \dot{V}(e) &= e^\top \left(\tau - C(q,\dot{q})\dot{q} - D(q,\dot{q})\dot{q} - G(q) - M(q)\ddot{q}_r \right) \\ &\quad + e^\top C(q,\dot{q}) e \\ &= e^\top \left(\tau - M(q)\ddot{q}_r - C(q,\dot{q})\dot{q}_r - D(q,\dot{q})\dot{q}_r - G(q) \right) \\ &\quad - e^\top D(q,\dot{q}) e \end{aligned}$$

Since we have assumed that M, C, D, and G are known, we can directly cancel these terms in the previous expression by choosing

$$\tau = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + D(q,\dot{q})\dot{q}_r + G(q) + \tau_c \qquad (4)$$

where τ_c is a control term to be specified later. This choice leads to

$$\dot{V}(e) = -e^\top D(q,\dot{q}) e + e^\top \tau_c \qquad (5)$$

Since D is positive definite by assumption, we know $\dot{V}$ is negative definite when $\tau_c = 0$, thus achieving asymptotic trajectory tracking. We can also choose τ_c to meet convergence rate requirements; one simple choice is $\tau_c = -Ke$ with $K \succ 0$.

This concludes the derivation of the trajectory controller without any communication delays. Our goal now is to design τ_c such that this controller still achieves trajectory tracking despite the addition of communication delays. In order to analyze the effect of delays, we will utilize dissipativity-based tools.

2.2 Dissipative Systems

The theory of dissipative systems originated in Willems (1972). Dissipativity can be seen as one of the possible generalizations of Lyapunov analysis to interconnected systems, and is a direct generalization of passivity. As with Lyapunov analysis, dissipativity-based analysis utilizes a notion of energy in a system as a function of its state, and as with passivity-based analysis, dissipativity-based analysis models the transfer of energy between interconnected systems, but differs in that it considers a more general definition of energy transfer.

The systems under consideration in this section are all of the form

$$\begin{array}{lll} \dot{x} = f(x,u) & x \in \mathcal{X} & \\ y = h(x,u) & y \in \mathcal{Y} & u \in \mathcal{U} \end{array} \tag{6}$$

where x is the state, u is the input, and y is the output of the system.

Definition A nonlinear system Σ of the form (6) is said to be *dissipative* with respect to some supply rate $s(t) = s\left(u(t), y(t)\right)$ if there exists a positive semi-definite storage function $V : \mathcal{X} \rightarrow \mathbb{R}_{\geq 0}$ such that

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(t)dt \tag{7}$$

along all feasible trajectories of x, for all $u \in U$, and for all $t_1 \geq t_0$. Σ is said to be *lossless* if the inequality in (7) is actually an equality and is said to be *strictly dissipative* if (7) is a strict inequality for all $t_2 > t_1$, except possibly when $V \equiv 0$. If V is differentiable with respect to time, then (7) becomes

$$\dot{V}(x(t)) \leq s(t) \tag{8}$$

Another useful equivalent definition of dissipativity is that for all x_0, there exists a lower bound $a = \mathbf{a}(x_0) \in \mathbb{R}$ such that:

$$\int_{t_0}^{t_1} s(t)dt \geq a \quad \forall u \; \forall t_1 \geq t_0 \tag{9}$$

In words, this means that an infinite amount of energy cannot be drawn out of a dissipative system.

Some of the most important properties of dissipative systems deal with multiple systems

Theorem 1. Given systems Σ_i for $i = 1, ..., k$, each dissipative with respect to corresponding supply rate $s_i(t)$, the combined system $\Sigma = \{\Sigma_i : i = 1, ..., k\}$ is dissipative with respect to the sum of the supply rates $s(t) = \sum_{i=1}^{k} s_i(t)$.

The proof of this theorem is straightforward, and is shown in Willems (1972). Also note that this theorem applies regardless of how the systems are interconnected. The real usefulness of this theorem comes when used when analyzing the stability of dissipative systems.

Theorem 2. Given a system Σ that is dissipative with respect to supply rate $s(t)$ with some associated storage function V, then Σ is stable (in the Lyapunov sense) in some neighborhood of the origin if V is locally positive definite and s is locally negative semi-definite.

This is just a rephrasing of Lyapunov's direct method. A similar result for asymptotic stability can be shown if Σ is strictly dissipative. We can combine the previous two theorems

Corollary 3. Given systems Σ_i for $i \in 1, ..., k$, each dissipative with respect to corresponding supply rate $s_i(t)$ with some associated storage function V_i, then combined system $\Sigma = \{\Sigma_i : i = 1, ..., k\}$ is stable if each V_i is locally positive definite and the sum of the supply rates is locally negative semi-definite.

We will use this corollary in following sections to derive sufficient conditions for stability of systems with communication delays.

3. DELAYS AS DISSIPATIVE SYSTEMS

We start by deriving a supply rate to which communication delays are dissipative and then show how this supply rate provides sufficient conditions for stability of systems with communication delays via Corollary 3.

3.1 Communication Delay Supply Rates

In many applications, communication delays are unknown and may vary in time in complex ways. Using the notation of Delay 1 in Fig. 1, we define a time varying delay system

$$\widehat{v}(t) = v(t - T(t)) \tag{10}$$

While we consider the length of the delay $T(t)$ to be unknown, we assume that

$$T(t) \geq 0 \tag{11}$$

$$\dot{T}(t) \leq 1 \tag{12}$$

The condition in (11) assures that the system is causal and the condition in (12) maintains that messages will arrive in the order in which they were transmitted.

Theorem 4. Time-varying delay systems as defined by (10) are dissipative with respect to

$$s(v, \widehat{v}) = \alpha(v) - \alpha(\widehat{v})\, c \tag{13}$$

where $\alpha : \mathcal{V} \rightarrow \mathbb{R}_{\geq 0}$ is some positive semi-definite function and $c \leq (1 - \dot{T}(t)) \; \forall t$.

Proof. The state of this system is all the values of the input v over the time interval starting at the current time and going back $T(t)$ seconds

$$x(t) = \{v(\tau) \mid \tau \in [t - T(t), t]\} \tag{14}$$

Now, we choose a positive semi-definite storage function V for this system and then try to find a supply rate s that meets the inequality in (8). A possible choice is

$$V(x(t)) = \int_{t-T(t)}^{t} \alpha(v(\tau))d\tau \tag{15}$$

Where $\alpha : \mathcal{V} \rightarrow \mathbb{R}_{\geq 0}$ is some positive semi-definite function. Using the Leibniz rule to differentiate

$$\begin{aligned}\dot{V}(x(t)) &= \alpha(v(t)) - \alpha(v(t-T(t)))\left(1-\dot{T}(t)\right)\\ &= \alpha(v(t)) - \alpha(\widehat{v}(t))\left(1-\dot{T}(t)\right)\\ &\leq \alpha(v(t)) - \alpha(\widehat{v}(t))\, c\end{aligned}$$

Note that this definition requires that $\dot{T}$ have an upper bound, but recall that it is already assumed in (12) to be bounded above by 1. If $\dot{T} \equiv 0$, then we can choose $c = 1$, in which case the above supply rate is lossless. If desired, we can also let c depend on time t.

Considering the system in Fig. 1 again, there are two delays, one in each direction of communication. It will be convenient for the coming analysis to combine the two delays into one system (dotted box in Fig. 1).

Corollary 5. The time-varying bi-directional delay system such as shown in Fig. 1 is dissipative with respect to $s_a(v,\widehat{w}) + s_b(w,\widehat{v})$ where

$$s_a(v,\widehat{w}) = \alpha_1(v) - \alpha_2(\widehat{w})\, c_2 \quad (16)$$
$$s_b(w,\widehat{v}) = \alpha_2(w) - \alpha_1(\widehat{v})\, c_1 \quad (17)$$

given $c_1 \leq (1-\dot{T}_1)$ and $c_2 \leq (1-\dot{T}_2)$, $\alpha_1 : \mathcal{V} \to \mathbb{R}_{\geq 0}$ and $\alpha_2 : \mathcal{W} \to \mathbb{R}_{\geq 0}$ are some positive semi-definite functions. The subscripts 1 and 2 on α and $\dot{T}$ represent the corresponding values for the Delay 1 and Delay 2 in Fig. 1.

Proof. From Theorem 4, we know that the delay systems are dissipative with respect to $s_1(v,\widehat{v})$ and $s_2(w,\widehat{w})$, respectively, where

$$s_1(v,\widehat{v}) = \alpha_1(v) - \alpha_1(\widehat{v})\, c_1$$
$$s_2(w,\widehat{w}) = \alpha_2(w) - \alpha_2(\widehat{w})\, c_1$$

From Theorem 1, the combined system is dissipative with respect to the sum of the supply rates

$$s_1(v,\widehat{v}) + s_2(w,\widehat{w}) = s_a(v,\widehat{w}) + s_b(w,\widehat{v})$$

We will see in the following sections that the structure of these supply rates makes it possible for us to verify if the overall system is stable by independently meeting dissipativity conditions for each system.

3.2 Stability Conditions

We now have everything needed to state sufficient conditions for stability of systems with communication delays

Corollary 6. Given a system setup as in Fig. 1, the combined system made up of the human operator, user interface, communication delay system, robot controller, and robot is stable if

(a) the operator / user interface system is dissipative with respect to $-s_a(v,\widehat{w})$ as defined in (16) and
(b) the robot control / robot system is dissipative with respect to $-s_b(w,\widehat{v})$ as defined in (17).

assuming that α_1 and α_2 are both defined to be locally positive definite.

Proof. We know from Corollary 5 that combined delay systems are dissipative with respect to $s_a(v,\widehat{w}) + s_b(w,\widehat{v})$, so given the conditions above, then the sum of all of the supply rates of the systems is equal to zero. Thus, from Corollary 3, the overall system is stable.

The requirement that α_1 and α_2 be locally positive definite ensures that every component of v and w remain bounded. The conditions above restrict the signals v and w to sublevel sets of α_1 and α_2, but if α_1 and α_2 are only semi-definite, the sublevel sets may not be bounded. However, this may be satisfactory if we can bound the other components of the state using invariance principles. This is the strategy we adopt in later sections.

Notice that if the delays increase so quickly that new messages are never received ($\dot{T}_i = 1$ and $c_i \leq 0$), then the stability conditions are the same as requiring that each subsystem be independently stable for any input. When new messages are received ($\dot{T}_i < 1$ and $c_i > 0$), the stability conditions are weaker than requiring that each subsystem be independently stable.

Also notice that these conditions were constructed so that each system needs to be dissipative with respect to a supply rate which is a function of locally available variables. This is very convenient as it allows us to design and analyze each system independently.

3.3 Choice of α_1 and α_2

We are free to choose any positive semi-definite α_1 and α_2 in the supply rates in (16) and (17), but some choices may make it easier than others to ensure dissipativity. In the following sections we use

$$\alpha_1(x_d) = \alpha_1\left(\begin{bmatrix} q_d \\ \dot{q}_d \end{bmatrix}\right) = ||\dot{q}_d + \lambda q_d||^2_{K_1} \quad (18)$$

$$\alpha_2(x) = \alpha_2\left(\begin{bmatrix} q \\ \dot{q} \end{bmatrix}\right) = ||\dot{q} + \lambda q||^2_{K_2} \quad (19)$$

where K_1, K_2 are positive semi-definite matrices and the norm $||q||_K = \sqrt{q^\top K q}$. This choice makes the design of the robot control much simpler, as it matches well with the definition of the tracking error in Section 2.1. This choice is also fine for the operator and UI, as we will show that for linear systems it is equivalent to a simpler supply rate.

4. USER INTERFACE ANALYSIS AND DESIGN

4.1 Human Pointing Dynamics

The user interface in our application involves a human operator controlling the position of a robot. If this position is specified by pointing, we can expect the dynamics to be consistent regardless of the specific interface being used. Humans have shown to generate similar motions when reaching and pointing with their arms, mouse pointers, laser pointers, and other devices. We use the Vector Integration to Endpoint (VITE) model, as first shown in Bullock and Grossberg (1988), to model pointing behaviors. In its simplest form, this model can be written as:

$$\begin{aligned}\dot{\nu} &= \gamma(-\nu + \zeta - p)\\ \dot{l} &= G[\nu]^+\end{aligned} \quad (20)$$

where l is the true position of the pointer, p is the feedback of the (perceived) position of the pointer, ζ is the target pointer position, ν is a state called the difference vector, G is a gain called the go signal, and γ is a system parameter. This model only represents single dimensional pointing motions, but it may be generalizable to two or three dimensions.

The VITE model describes a single motion made by a human trying to drive the true position of the pointer l to the target pointer position ζ. If there is overshoot, and the true pointer position l passes over the target pointer position ζ, the human stops quickly, but no corrective action is taken to move back closer to the target. The work by Beamish et al. (2006) showed that because of the feedback delays between the true position l and the perceived position p of the pointer, there is an inherent trade-off between the overshoot of the target and the time taken to reach the target that matches with Fitts's law, one of the most famous invariants for reaching and pointing motions. It was also shown that the overshoot of the target occurs whether $\nu(t) < 0$ for some t. In the next section, we will show that the the VITE model can be seen as a switched linear system, where the switching occurs when ν crosses zero, and it is dissipative only when $\nu \geq 0$ and there is no overshoot.

4.2 Dissipativity of Human Pointing

Our goal now is to determine when the user interface combined with the human operator is dissipative with respect to $-s_a(v, \widehat{w})$. We will see that by representing the VITE model as a switched linear system, we can show that it is dissipative as long as the operator does not overshoot their target. First, we consider the dissipativity of an arbitrary linear system.

Theorem 7. A linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

is dissipative with respect to $s(u, y) = ||u||^2_{K_1} - ||y||^2_{K_2}$ for positive semi-definite matrices K_1, K_2 if there exists a positive semi-definite solution P to the Riccati equation $PA + A^\top P + PBK_1^{-1}B^\top P + C^\top K_2 C = 0$.

Proof. Given there exists a $P \succeq 0$ such that $PA + A^\top P + PBK_1^{-1}B^\top P + C^\top K_2 C = 0$, try the storage function

$$\begin{aligned}
V(x) &= x^\top P x \\
\dot{V}(x) &= x^\top (PA + A^\top P)x + 2u^\top B^\top P x^\top \\
&= x^\top (PA + A^\top P)x + u^\top K_1 u + x^\top PBK_1^{-1}B^\top P x \\
&\quad - ||K_1^{-1}B^\top P x - u||^2_{K_1} \\
&\leq x^\top (PA + A^\top P + PBK_1^{-1}B^\top P)x + u^\top K_1 u \\
&= -x^\top C^\top K_2 C x + u^\top K_1 u \\
&= -||y||^2_{K_2} + ||u||^2_{K_1}
\end{aligned}$$

This theorem will let us determine when the VITE model is dissipative. Referring back to Section 3.3, the supply rate in this theorem is not quite of the form that we want as in (18) and (19); we are using $s = ||u||^2_{K_1} - ||y||^2_{K_2}$ instead of $s = ||\dot{u} + \lambda u||^2_{K_1} - ||\dot{y} + \lambda y||^2_{K_2}$. We will see in the next section in the design of the robot controller that the latter supply rate is much easier to work with. Before then, we can show that linear systems, the two supply rates can be tested for with the same conditions, the solution to the Riccati Equation.

Theorem 8. A linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

is dissipative with respect to $s(u, y) = ||\dot{u} + \lambda u||^2_{K_{1,i}} - ||\dot{y} + \lambda y||^2_{K_{2,i}}$ for positive semi-definite matrices K_1, K_2 if there exists a positive semi-definite solution P to the Riccati equation $PA + A^\top P + PBK_1^{-1}B^\top P + C^\top K_2 C = 0$.

Proof. Construct an augmented system using the definitions:

$$\widetilde{x} = \dot{x} + \lambda x \qquad \widetilde{y} = \dot{y} + \lambda y \qquad \widetilde{u} = \dot{u} + \lambda u$$

With this, the dynamics become

$$\begin{aligned}
\dot{\widetilde{x}} &= \ddot{x} + \lambda \dot{x} \\
&= A\dot{x} + B\dot{u} + Ax + Bu \\
&= A\widetilde{x} + B\widetilde{u} \\
\widetilde{y} &= C\dot{x} + Cx \\
&= C\widetilde{x}
\end{aligned}$$

Thus, by construction, the augmented system is consistent with the original system's dynamics. Using Theorem 7, if there exists a $P \succeq 0$ such that $PA + A^\top P + PBK_1^{-1}B^\top P + C^\top K_2 C = 0$, then the system is dissipative with respect to $s(\widetilde{u}, \widetilde{y}) = ||\widetilde{u}||^2_{K_1} - ||\widetilde{y}||^2_{K_2} = ||\dot{u} + \lambda u||^2_{K_1} - ||\dot{y} + \lambda y||^2_{K_2}$

We can now use this theorem to analyze the dissipativity of the VITE model.

Theorem 9. The VITE model in (20) is dissipative with respect to

$$s\left(\begin{bmatrix} p \\ \dot{p} \end{bmatrix}, \begin{bmatrix} l \\ \dot{l} \end{bmatrix}\right) = ||\dot{p} + \lambda p||^2_{K_1} - ||\dot{l} + \lambda l||^2_{K_2} \qquad (21)$$

when $\nu(t) \geq 0$ for all t.

Proof. The VITE model can be seen as a switched linear system, where the switching occurs when $\nu = 0$. For $\nu(t) \geq 0$, we can define

$$x = \begin{bmatrix} \nu \\ l \end{bmatrix} \qquad u = \begin{bmatrix} p \\ \zeta \end{bmatrix} \qquad y = l$$
$$A = \begin{bmatrix} -\gamma & 0 \\ G & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -\gamma & \gamma \\ 0 & 0 \end{bmatrix}$$
$$C = [0 \ \ 1]$$

The controllability and observability matrices for this system are

$$\begin{aligned}
\mathcal{C} &= [B \ \ AB] \\
&= \begin{bmatrix} -\gamma & \gamma & \gamma^2 & -\gamma^2 \\ 0 & 0 & -\gamma G & \gamma G \end{bmatrix} \\
\mathcal{O} &= \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ G & 0 \end{bmatrix}
\end{aligned}$$

both of which are full rank, so the system is controllable and observable and thus has a solution to the Riccati equation. So by Theorem 8, it is dissipative with respect to the supply rate in (21).

Theorem 10. The VITE model in (20) is not dissipative with respect to the supply rate in (21) when $\nu(t) < 0$ for all t.

Proof. When $\nu(t) < 0$, then y is a constant and $\dot{y} = 0$. If we let $c = ||\dot{y} + \lambda y||^2_{K_2} = ||\lambda y||^2_{K_2}$

$$\int_{t_0}^{t_1} s(t)dt = \int_{t_0}^{t_1} \left(||\dot{u} + \lambda u||^2_{K_1} - ||\dot{y} + \lambda y||^2_{K_2}\right) dt$$
$$= \int_{t_0}^{t_1} \left(||\dot{u} + \lambda u||^2_{K_1} - c\right) dt$$

Choosing $u \equiv 0$

$$\int_{t_0}^{t_1} s(t)dt = -c(t_1 - t_0)$$

which goes to $-\infty$ as $t_1 \to \infty$. From the alternative definition of dissipativity in (9), this means that the system is not dissipative with respect to this supply rate.

We can see that the operator's behavior is dissipative when there is no overshoot. There are two possibilities to ensure dissipativity, either prevent overshoot or add dissipation when there is overshoot.

5. ROBOT CONTROLLER ANALYSIS AND DESIGN

The last step in our design is ensure that the robot control system meets the stability conditions and maintains asymptotic trajectory tracking. We will find that while the dissipativity conditions are enough to ensure that the tracking error is stable, we will require something additional to ensure asymptotically stable error.

Theorem 11. Given

$$\tau_c = \begin{cases} -K_d\left[(\dot{q} + \hat{\dot{q}}_d) + \lambda(q + \hat{q}_d)\right] & \text{if } s_b(x, \hat{x}_d) \geq e^\top K_e e \\ -K_e e & \text{otherwise} \end{cases} \tag{22}$$

with K_d, $(c_1 K_1 - K_d)$, $(K_d - K_2)$ positive semi-definite and with K_e positive definite, $c_1 \leq 1 - \dot{T}_1$, and

$$\alpha_1(\hat{x}_d) = ||\hat{\dot{q}}_d + \lambda \hat{q}_d||^2_{K_1}$$
$$\alpha_2(x) = ||\dot{q} + \lambda q||^2_{K_2}$$

then the vehicle/on-board control system is dissipative with respect to $-s_b(x, \hat{x}_d) = -\alpha_2(x) + \alpha_1(\hat{x}_d)\, c_1$ and achieves asymptotic trajectory tracking.

Proof.

This controller switches between ensuring dissipativity and ensuring error convergence. We can show that

$$\dot{V}(e) \leq \min\{-s_b(x, \hat{x}_d), -e^\top K_e e\}$$

since we know from (5) that

$$\dot{V}(e) = -e^\top D(q, \dot{q})e + e^\top \tau_c \leq e^\top \tau_c$$

Thus, we want

$$e^\top \tau_c \leq \min\{-s_b(x, \hat{x}_d), -e^\top K_e e\}$$

We can see that $\tau_c = -K_d\left[(\dot{q} + \hat{\dot{q}}_d) + \lambda(q + \hat{q}_d)\right]$ implies

$$e^\top \tau_c = -\left[(\dot{q} - \hat{\dot{q}}_d) + \lambda(q - \hat{q}_d)\right]^\top K_d\left[(\dot{q} + \hat{\dot{q}}_d) + \lambda(q + \hat{q}_d)\right]$$
$$= -\left[(\dot{q} + \lambda q) - (\hat{\dot{q}}_d + \lambda \hat{q}_d)\right]^\top K_d\left[(\dot{q} + \lambda q) + (\hat{\dot{q}}_d + \lambda \hat{q}_d)\right]$$
$$= -\left[||\dot{q} + \lambda q||^2_{K_d} - ||\hat{\dot{q}}_d + \lambda \hat{q}_d||^2_{K_d}\right]$$
$$\leq -||\dot{q} + \lambda q||^2_{K_2} + ||\hat{\dot{q}}_d + \lambda \hat{q}_d||^2_{K_1} c_1$$
$$= -s_b(x, \hat{x}_d)$$

And clearly, $\tau_c = -K_e e$ implies $e^\top \tau_c \leq -e^\top K_e e$, so by switching when $s_b(x, \hat{x}_d)$ crosses $e^\top K_e e$, we meet both conditions and

$$\dot{V}(e) \leq \min\{-s_b(x, \hat{x}_d), -e^\top K_e e\}$$

6. DESIGN SUMMARY

We have derived all of the components of our controller design. Given a system as in Fig. 1 with the robot system modeled by (1) and delays each modeled by (10), then if we design the robot control system according to (4) and (22) with a human operator that does not overshoot their target, then the overall system is stable and achieves asymptotic trajectory tracking $y(t) \to \hat{v}(t)$ as $t \to \infty$.

Stability is ensured because a system designed this way meets the conditions of Corollary 6. One technical point is that α_1 and α_2 as we defined them are only positive definite with respect to $(\dot{q}_d + \lambda q_d)$ and $(\dot{q} + \lambda q)$, respectively, so Corollary 6 only implies that these terms are stable; however, we can use invariance principles to also show that the terms q_d, $\dot{q}_d$, q, and $\dot{q}$ are each individually stable. While the overall system should not be asymptotic stable, as this would mean $x_d \to 0$, we have already shown that e is asymptotically stable to the origin for this choice of switching control.

7. SIMULATION

Using the design proposed in this paper, we simulated a robotic vehicle being controlled remotely over a delayed communication link by a simulated human operator. This simulation shows that our design behaves as we expect: the state of the system is stable and asymptotic trajectory tracking is achieved despite the time varying delays. For the vehicle, we used a model of a small unmanned underwater vehicle (UUV), for which $q = [\text{Longitude}, \text{Latitude}, \text{Depth}, \text{Roll}, \text{Pitch}, \text{Yaw}]^\top$. For the human operator, we used the VITE model with a constant target pointer position $\zeta = 1$ and with parameters $\gamma = 1$ and $G = 0.2$, which ensured that there was no overshoot. The user interface was configured to control just one component of the state of the robot, the latitude, while the other components were set at a target of zero. The communication delays were chosen as $T_1(t) = T_2(t) = 0.5 + 0.5 * \sin(0.1\, t)$ and the other parameters for the design were set as $K_1 = 10$ I, $K_2 = K_d = 8$ I, and $K_e = 10$ I, which meets all of the conditions specified in the design.

Fig. 2 shows two different views of the trajectory of the vehicle over time. Fig. 3 shows the magnitude of the error $||e||$ over time. Fig. 4 shows each of the elements of the x over time compared to the desired state value x_d. With these, we can see that the error is asymptotically stable, and for these choices of gains, mostly converges before the human reaches their target.

8. CONCLUSION

We have demonstrated an application of dissipativity theory to the analysis and design of systems with communication delays. We used an explicit model for the human

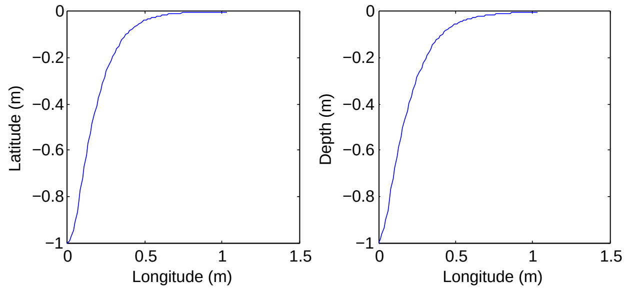

Fig. 2. The trajectory of the vehicle.

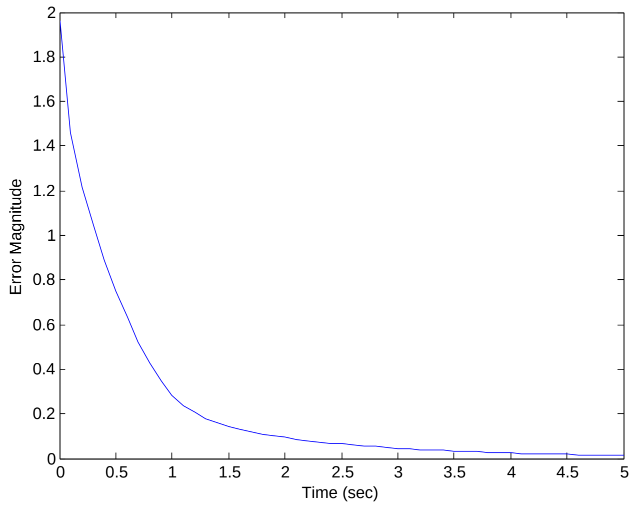

Fig. 3. Magnitude of the trajectory error $||e||_2$ over time.

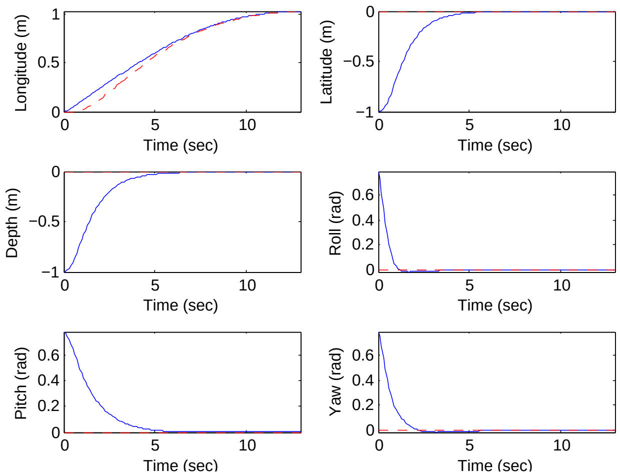

Fig. 4. Each of the components of the state over time. The solid blue lines show true values of state x and dotted red lines show the desired state values $\widehat{x}_d$

operator's behavior and robot's dynamics. Our dissipativity methods allowed us to analyze and design these systems separately. Natural extensions to this work include an UI design that is always dissipative, multi-dimensional human model, non-holonomic robot model, adaptive versions of the controller designs, and different types of communication links, including packet-based communication.

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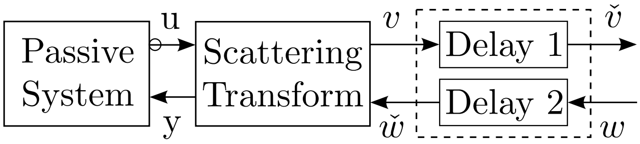

Fig. A.1. Scattering transform applied between a passive system and a communication delay system.

Appendix A. CONNECTION TO PASSIVITY AND THE SCATTERING TRANSFORM

As mentioned in Section 1, the scattering transform is a tool frequently used to compensate for unknown communication delays. The scattering transform for time-varying delays is defined by

$$v = \frac{1}{\sqrt{2b}}(\mathrm{y} + b\mathrm{u}) \qquad \sqrt{c_2}\widehat{w} = \frac{1}{\sqrt{2b}}(\mathrm{y} - b\mathrm{u}) \qquad \text{(A.1)}$$

where b is some positive constant, c_2 is a non-negative gain, $\mathrm{y}, \mathrm{u} \in \mathbb{R}^n$, and the other variables are as labeled in Fig. A.1.

In much of literature on the scattering transform, humans using a haptic feedback controller are assumed to behave passively, i.e. dissipative with respect to $s(u, y) = \mathrm{u}^\top \mathrm{y}$. This passivity assumption is convenient because when the scattering transform is connected in negative feedback to a passive system, it meets the stability conditions in Corollary 6 (a) when $\alpha_1(\cdot) = \alpha_2(\cdot) = \frac{1}{2}||\cdot||^2$.

Theorem 12. The scattering transform is lossless with respect to

$$s(t) = \mathrm{u}^\top \mathrm{y} - [\alpha_1(v) - \alpha_2(\widehat{w})c_2]$$

where $\alpha_1(\cdot) = \alpha_2(\cdot) = \frac{1}{2}||\cdot||^2$ and where $c_2 \leq (1 - \dot{T}_2)$.

Proof. Solving for u and y

$$\mathrm{u} = \sqrt{\frac{1}{2b}}\; v - \sqrt{\frac{c_2}{2b}}\; \widehat{w}$$

$$\mathrm{y} = \sqrt{\frac{b}{2}}\; v + \sqrt{\frac{c_2 b}{2}}\; \widehat{w}$$

Combining these terms

$$\begin{aligned} \mathrm{u}^\top \mathrm{y} &= \frac{1}{2}||v||^2 - \frac{1}{2}||\widehat{w}||^2 c_2 \\ &= \alpha_1(v) - \alpha_2(\widehat{w})c_2 \end{aligned}$$

We can choose $V = 0$, in which case

$$\dot{V} = 0 = \mathrm{u}^\top \mathrm{y} - [\alpha_1(v) - \alpha_2(\widehat{w})c_2]$$

From this we can see that scattering transform is one way of transforming a passive system into a system that is dissipative with respect to a supply rate compatible with delays.