Tracking Performance under Time Delay and Asynchronicity in Distributed Camera Systems

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***Abstract*— Distributed camera systems are typically used to simultaneously track multiple targets. Communication between cameras enables the ability to monitor targets in a complex environment where occlusion happens. Asynchronicity and time delay are among the most important factors that affect tracking error. We provide a feedback law controlling the pan and tilt of each camera to track the centroid of a cluster of point targets in the field of view. Assuming that the estimates for target motion are available only infrequently and asynchronously when occlusion happens, we compute a worst-case lower bound on the frequency for exchanging estimates between cameras. Our results may help camera system designers to determine the response time and tracking ability for distributed camera systems.**

I. Introduction

Recent advancements in technology have dramatically reduced the cost of manufacturing and installing distributed camera systems. Research in image processing and computer vision has enabled using cameras to track moving targets with high accuracy in real time. Distributed camera systems have the advantage of cooperatively tracking targets even when occlusion happens, i.e. when the field of view (FOV) of some cameras is blocked by obstacles. By communication, cameras with occlusion may obtain estimates of the blocked targets from other cameras monitoring the targets from different viewing angles. Communication delays and asynchronicity exist in most such distributed systems [1]. Controller design under time-delay has been a long lasting focus in the control literature c.f. [2]–[5]. The problem of asynchronicity has gained much recent interest due to the research thrusts in cooperative control and sensor networks [6]–[8].

There have been ongoing efforts to generalize Lyapunov stability theory to systems with hybrid nature [9]–[13]. In this paper, we develop methods based on input to state stability (ISS) for discrete-time systems [14]–[17] and apply them to a simplified perspective dynamic model [18]–[20] of the distributed camera systems to establish relationship between tracking error, delay, and asynchronicity. By explicitly determining an ISS Lyapunov function for the camerat racking system, one can estimate the size of a neighborhood to which the state will converge in equilibrium. This gives a measure of the system performance for different possible values of comunication/computation delays and different control gains. Such a metric is highly desirable in applications involving control/scheduler co-design; if a certain level of tracking performance is required, we can schedule the state estimation/computation of the control law so that the control interval never exceeds a deadline, which we choose together with control gain and inter-measuremnt interval in such a way that the performance requirements are met.

In section II, we briefly review the perspective dynamics that model the motion of a point target in the image plane of a camera. We then introduce tracking controllers for a single camera to track one or a cluster of point targets in section III. In section IV, we consider the effect of time delay and asynchronicity on the tracking performance for two cameras tracking multiple targets. Simulation results will be presented in section V.

II. The Perspective Dynamical System

We consider cameras with controllable pan and tilt. We first introduce a model that describes the motion of a point target in the image plane of a camera.

Consider a single camera observing the space $\mathcal{R}^3$. We may establish the camera coordinate frame using three orthogonal unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ defining the three axes. It is required that the origin is at the lens of the camera, and $\mathbf{i}$ and $\mathbf{j}$ span a plane that is parallel to the image plane of the camera. Hence $\mathbf{k}$ is perpendicular to the image plane. Rotation around $\mathbf{i}$ is the tilt motion and rotation around $\mathbf{j}$ is the pan motion. Since a camera is often fixed in location. We assume that the origin of the inertial frame coincides with the origin of the camera frame. When the pan and tilt motion are performed, the body coordinate frame rotates with the camera. The orientation of the camera in the inertial frame can be described by a 3×3 matrix g that belongs to the special orthogonal group $SO(3)$ i.e. $gg^T = I$.

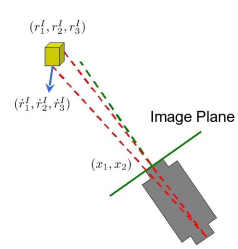

Fig. 1. Schematic of the perspective system

Let $\mathbf{r}_I$ represent a point in $\mathcal{R}^3$ (see fig. 1). The position of the point in the camera's coordinate frame is $\mathbf{r} = g^{-1}\mathbf{r}_I$. Taking time derivatives on both sides of $\mathbf{r}_I = g\mathbf{r}$, and

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considering that $\dot{g} = g\hat{\Omega}$ where $\hat{\Omega}$ is a 3×3 skew symmetric matrix, we obtain $\dot{\mathbf{r}}_I = g(\hat{\Omega}\mathbf{r}+\dot{\mathbf{r}})$. The $\hat{\Omega}$ here is also a matrix representation of the angular velocity vector Ω of the camera. This angular velocity can be written as $\Omega = u\mathbf{i} + v\mathbf{j} + 0\mathbf{k}$ where u and v stand for the pan and tilt controls.

If we define a velocity vector $\mathbf{b} = g^{-1}\dot{\mathbf{r}}_I$, then we have $\dot{\mathbf{r}} = -\Omega \times \mathbf{r} + \mathbf{b}$. Now consider the projection of the vector $\mathbf{r}$ onto the image plane of the camera with distance equal to unit length to the lens. We can write $\mathbf{r} = r_1\mathbf{i}+r_2\mathbf{j}+r_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$. Let the projection of $\mathbf{r}$ be described as $\mathbf{x} = x_1\mathbf{i} + x_2\mathbf{j}$. One has the following relationships between (r_1, r_2, r_3) and (x_1, x_2):

$$x_1 = \frac{r_1}{r_3} \text{ and } x_2 = \frac{r_2}{r_3} \tag{1}$$

The differential equations that (x_1, x_2) satisfy are called the *perspective dynamics*. We now derive the perspective dynamical system assuming that b_1, b_2, and b_3 are constant. First, $\dot{\mathbf{r}} = -\Omega \times \mathbf{r} + \mathbf{b}$ implies that

$$\begin{aligned} \dot{r}_1 &= b_1 - r_3 v \\ \dot{r}_2 &= b_2 + r_3 u \\ \dot{r}_3 &= b_3 - r_2 u + r_1 v. \end{aligned} \tag{2}$$

We also define (B_1, B_2, B_3) as

$$B_1 = \frac{b_1}{r_3}, \quad B_2 = \frac{b_2}{r_3}, \quad \text{and } B_3 = \frac{b_3}{r_3}. \tag{3}$$

Taking the time derivatives of $x_1 = \frac{r_1}{r_3}$, we have,

$$\dot{x}_1 = -\frac{r_1\dot{r}_3}{r_3^2} + \frac{\dot{r}_1}{r_3} = x_1x_2u - (x_1^2+1)v + B_1 - B_3x_1. \tag{4}$$

Taking the time derivatives of $x_2 = \frac{r_2}{r_3}$ yields

$$\dot{x}_2 = -\frac{r_2\dot{r}_3}{r_3^2} + \frac{\dot{r}_2}{r_3} = (x_2^2+1)u - x_1x_2v + B_2 - B_3x_2. \tag{5}$$

Here, x_1, x_2, B_1, and B_2 are the position and velocity of the projected point in the image plane. They are directly measured by the camera. But B_3 can not be directly observed.

In the target tracking problem, we want to control the value of x_1 and x_2 so that the target is always in the field of view. If the camera is far away from the target, a reasonable assumption to make in this case is that B_3 is sufficiently small to be neglected. Since B_1 and B_2 can be directly observed, we can study the subsystem system formed by (x_1, x_2) as

$$\begin{aligned} \dot{x}_1 &= x_1x_2u - (x_1^2+1)v + B_1 \\ \dot{x}_2 &= (x_2^2+1)u - x_1x_2v + B_2. \end{aligned} \tag{6}$$

In the following sections, we design controllers for the simplified perspective dynamics (6).

III. Controller design for Single Camera

In this section, we study the problem of controling a single camera to track multiple targets in continuous time. When there is only one target to track, we design a controller for a single camera system to keep the target at the center of the FOV. When there are multiple targets, the controller keeps the centroid of the targets at the center of the FOV.

A. *Tracking multiple targets with single camera*

Suppose there are N targets, where $N \geq 1$. Define the centroid of all targets as (x_{c1}, x_{c2}) where $x_{c1} = \frac{1}{N}\sum_{i=1}^N x_{i1}$ and $x_{c2} = \frac{1}{N}\sum_{i=1}^N x_{i2}$, and i is the index for the targets. Then the equations for $\dot{x}_{c1}$ and $\dot{x}_{c2}$ are

$$\begin{aligned} \dot{x}_{c1} &= \frac{1}{N}(\sum_{i=1}^N x_{i1}x_{i2}u - \frac{1}{N}\sum_{i=1}^N(x_{i1}^2+1)v + \frac{1}{N}\sum_{i=1}^N B_{i1} \\ \dot{x}_{c2} &= \frac{1}{N}\sum_{i=1}^N(x_{i2}^2+1)u - \frac{1}{N}\sum_{i=1}^N x_{i1}x_{i2}v + \frac{1}{N}\sum_{i=1}^N B_{i2}. \end{aligned} \tag{7}$$

We now define the matrix

$$G_c = \frac{1}{N}\begin{bmatrix} \sum_{i=1}^N x_{i1}x_{i2} & \sum_{i=1}^N(x_{i1}^2+1) \\ \sum_{i=1}^N(x_{i2}^2+1) & \sum_{i=1}^N x_{i1}x_{i2} \end{bmatrix}. \tag{8}$$

One can verify that $\det(G_c) > 0$ by the induction method. Therefore, the control law can be obtained by

$$\begin{bmatrix} u \\ v \end{bmatrix} = G_c^{-1}\left(A\begin{bmatrix} x_{c1} \\ x_{c2} \end{bmatrix} - \frac{1}{N}\begin{bmatrix} \sum_{i=1}^N B_{i1} \\ \sum_{i=1}^N B_{i2} \end{bmatrix}\right) \tag{9}$$

where A is a 2×2 Hurwitz matrix. In the idealized case that all the states are known exactly, the equilibrium state (which corresponds to the centroid being at the center of the FOV) is globally asymptotically stable.

Occlusion happens when one or more targets are blocked by obstacles in the FOV of one camera. In this case, communication between cameras is necessary for tracking the targets. For example, every camera in the distributed camera system developed in [1] has the ability to request estimates about the blocked targets from another camera that has clear view of the targets. The estimates obtained from other cameras suffer time-delay and asynchronicity. We investigate how these factors affect the tracking error.

A typical pan and tilt camera has embedded computers that compute the control actions. Therefore, control of the cameras happen discretely. We discretize the perspective dynamical system (6) under the assumption that the control for pan and tilt happens accurately at time instants t_k where $k = 0, 1, 2, ...,$ and $\Delta t = t_k - t_{k-1}$ is constant for all k. Let $G_k = G(t_k)$ and define

$$F_k = \begin{bmatrix} B_1(t_k) \\ B_2(t_k) \end{bmatrix}, \ \mathbf{u}_k = \begin{bmatrix} u(t_k) \\ v(t_k) \end{bmatrix}. \tag{10}$$

Then the discrete perspective dynamics for one camera are

$$\mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + G_k\mathbf{u}_k\Delta t + F_k\Delta t. \tag{11}$$

B. Tracking a single occluded target with delay

Consider the following setup. Camera C1 is tracking a single target; the goal is to maintain the target as close as possible to the center of the FOV. The pan/tilt control $\mathbf{u}$ for the camera is calculated with an interval of T seconds, and simultaneously with the release of each control command, the camera accepts a measurement of the image-plane position x and velocity F of the target. Suppose now that the target is occluded in the FOV of C1. Camera C1 then requests estimates of the state of the occluded target from other cameras in the distributed system. Due to communication delay and necessity of converting the estimates from the measuring camera's reference frame to the reference frame of C1, the state measurements $x(t_k)$ and $F(t_k)$ at C1 are available with some delay. Owing to the finite time to compute the control law, the corresponding pan/tilt control is available at time $t_{k+m} = t_k + mT$, where m is a positive integer, $m > 1$. In practical situations, both m and T may vary; we assume here that both are constant and known, and represent the worst-case delay.

If we could measure the state and compute the corresponding control effort without computation delay, the discretized control law for tracking a single target at time t_j would be

$$\mathbf{u}_j = G_j^{-1}(Ax(t_j) - F_j) \tag{12}$$

Due to computation delay and communication delays when the target becomes occluded, however, our control effort calculations at time t_j must rely on estimates of $x(t_j)$ based on measurements made at some previous time, t_k, $k < j \leq k + m$. A simple estimator can be constructed to obtain the estimates recursively as follows:

$$\hat{x}(t_{j+1}) = x(t_j) + G_j\mathbf{u}_jT + F_jT \tag{13}$$

$$\hat{F}_{j+1} = \hat{F}_j \tag{14}$$

$$\mathbf{u}_j = \hat{G}_j^{-1}(A\hat{x}(t_j) - \hat{F}_j) \tag{15}$$

where $x(t_k)$ and F_k are the measured values of the target's position and velocity, respectively, and are assumed to be known exactly. The hatted values represent estimates, based on the previous available measurement. If at time t_j a measurement of the state is not available, the $\hat{x}$ and $\hat{F}$ are used instead of the exact values, so that $x = \hat{x}$ and $F = \hat{F}$ in the above equations.

If the estimates in (13)-(15) are used, then the value of $\hat{x}$ at time t_{k+1} will be (assuming that a measurement is made at time t_k):

$$\hat{x}(t_{k+1}) = x(t_k) + G_k(\hat{G}_k^{-1}(A\hat{x}(t_k) - \hat{F}_k))T + F_kT \tag{16}$$

We write $\hat{x}(t_k)$ as $\hat{x}(t_k) = x(t_k) + \xi_k$, where ξ_k is the state estimation error. We can also define a velocity estimation error, ξ_k^F, where $\hat{F}_k = F_k + \xi_k^F$. These errors usually grow with the length of the inter-measurement time mT. In this paper, we assume that the errors in estimates of the target's position and velocity, ξ_k and ξ_k^F, respecively, are independent random variables, and furthermore that ξ_k is uniformly distributed over the set $\mathcal{X} = \{\,\phi : \|\,\phi\,\| \leq L(\Delta t)^q\,\}$, while ξ_k^F is uniformly distributed over $\mathcal{XF} = \{\,\psi : \|\,\psi\,\| \leq L(\Delta t)^{q-1}\,\}$. Here $L \in \mathbb{R}^+$ and $q \in \mathbb{Z}^+$ are known, and Δt is the time since the last available measurement.

We pick A to be a diagonal matrix, $A = -K_kI$, where I is the 2×2 identity matrix, and K_k is a positive scalar. This means that K_k may be interpreted as an adjustable control gain. Minimizing $\|\,\hat{x}(t_{k+1})\,\|$ with respect to K_k in eq. (13), we see that we should set $K_k = \frac{1}{T}$. In fact, if we assumed that estimation errors are negligible, this control gain would bring x to $x = 0$ in a single discrete-time step. However, since T is by assumption small, a control gain of $\frac{1}{T}$ may be physically impossible to implement. We therefore set $K_k = \frac{1}{\alpha T}$, where $\alpha > 1$, so that the control $\mathbf{u}_k = -K_kI\hat{x}(t_k)$ does not exceed actuator limits.

We will now show that the tracking system described by (16), with A defined as above, is discrete-time input to state stable (ISS); that is, the state x converges to a small neighbourhood of the center of C1's FOV. The size of this neighbuorhood is determined by the magnitude of the estimation error in the position and velocity of the target.

For $\xi \in \mathcal{X}$, it can be shown (through simulation) that error due to imperfect knowledge of the state $x(t_k)$ in calculating matrix $\hat{G}$ is in fact negligible and may be absorbed into the estimation error of $x(t_{k+1})$. Then (16) can be rewritten into the following perturbed equations:

$$x(t_{k+1}) = (I + AT)x(t_k) + \tilde{\mathbf{u}}_k \tag{17}$$

$$\tilde{\mathbf{u}}_k = AT\xi_k + \xi_k^FT \tag{18}$$

For any small fixed T, this can be interpreted as a discrete-time plant with open-loop dynamics described by $x(t_{k+1}) = (I+AT)x(t_k)$, and a bounded stochastic input $\tilde{\mathbf{u}} = AT\xi_k + \xi_k^FT$. Note that $(I+AT)$ is a Schur matrix. It is a well-known result that a stable linear system with bounded input is ISS (See, for examlple, [14]). We will show that the ISS property holds even under long measurement times, and calculate an upper bound on the expected tracking performance of the system.

We define the following quadratic Lyapunov function:

$$V(x(t)) = \frac{1}{2}x(t)'x(t) \tag{19}$$

In the following, the abridged notation V_k is used for $V(x(t_k))$, and $\hat{V}_k$ for $\hat{V}(x(t_k))$. We are interested in the behavior of our system at the measurement instants, $[t_k, t_{k+m}, t_{k+2m}...]$; we want to estimate the upper bound for the steady state value of the Lyapunov function, which is used to measure tracking performance.

The following two lemmas are used repeatedly in the proofs in this paper:

Lemma 3.1: Given a quadratic polynomial

$$p(x) = -ax^2 + bx + c$$

with $a, b, c \in \mathbb{R}^+$, $a \neq 0$, and a non-zero affine function

$$q(x) = dx + f$$

with $d, f \in \mathbb{R}^+$, there exists some $C > 0$ such that

$$(p + q)(x) < 0 \text{ for all } x > C$$

.

Proof: Let

$$C = \frac{(b+d) + \sqrt{(b+d)^2 + 4a(c+f)}}{2a} > 0$$

Then, $(p+q)(C) = 0$, and $\frac{d}{dx}(p+q) = (-2ax + (b+d))$ is strictly negative for all $x \geq C$. Therefore for $x > C$, $(p+q)(x) < 0$. ■

Lemma 3.2: Given an inequality $r(x) \leq s(x)$ for all $x > 0$, with

$$s(x) = ax^2 + bx + c$$

where $a, b, c \in \mathbb{R}^+$, $a \neq 0$, and $s(x = \phi) = \Phi$ for some $\phi > 0$, it follows that $r(x) \leq \Phi$ for all $0 < x \leq \phi$.

Proof: For all $x > 0$, $\frac{d}{dx}s(x) = 2ax + b > 0$. Therefore, $s(x)$ is a strictly increasing function of x for all $x > 0$, and it follows that $r(x) \leq s(x) \leq \Phi$ for all $0 < x \leq \phi$. ■

We now turn to our camera tracking system:

Lemma 3.3: For the system defined by (17)-(18) and discrete ISS Lyapunov function (19), $V_{k+m} - V_k < 0$ whenever $\| x_c(t_j) \| > h$ for some constant h.

Proof: The difference $V_{k+m} - V_k$ can be written as

$$V_{k+m} - \hat{V}_{k+m} + \hat{V}_{k+m} - V_k$$

and $V_{k+m} - \hat{V}_{k+m}$ and $\hat{V}_{k+m} - V_k$ can then be bounded individually.

Begin by finding an upper bound on $\hat{V}_{k+m} - V_k$. First note that, for all i, j, we can write:

$$\begin{aligned} V_j - V_i &= \frac{1}{2}x(t_j)'x(t_j) - \frac{1}{2}x(t_i)'x(t_i) \\ &= x(t_i)'(x(t_j) - x(t_i)) \\ &\quad + \frac{1}{2}(x(t_j) - x(t_i))'(x(t_j) - x(t_i)) \end{aligned}$$

Using this form, we see that $\hat{V}_{k+1} - V_k$ satisfies the following equation:

$$\begin{aligned} \hat{V}_{k+1} - V_k &= x(t_k)'(-K_kT\hat{x}(t_k) + (F_k - \hat{F}_k)T) \\ &\quad + \frac{1}{2}\left\| -K_kT\hat{x}(t_k) + (F_k - \hat{F}_k)T \right\|^2 \\ &= x(t_k)'\left(-\frac{\hat{x}(t_k)}{\alpha} + \xi_k^F T\right) \\ &\quad + \frac{1}{2}\left\| -\frac{\hat{x}(t_k)}{\alpha} + \xi_k^F T \right\|^2 \\ &= x(t_k)'\left(-\frac{x(t_k)}{\alpha} + \frac{\xi_k}{\alpha} + \xi_k^F T\right) \\ &\quad + \frac{1}{2}\left\| -\frac{x(t_k)}{\alpha} + \frac{\xi_k}{\alpha} + \xi_k^F T \right\|^2 \\ &= -\frac{1}{\alpha}x(t_k)'x(t_k) + \frac{1}{\alpha}x(t_k)'\xi_k + x(t_k)'\xi_k^F T \\ &\quad + \frac{1}{2}\frac{1}{\alpha^2}x(t_k)'x(t_k) - \frac{1}{2}\frac{2}{\alpha}x(t_k)'(\frac{\xi_k}{\alpha} + \xi_k^F T) \\ &\quad + \frac{1}{2}(\frac{\xi_k}{\alpha} + \xi_k^F T)'(\frac{\xi_k}{\alpha} + \xi_k^F T) \\ &= \left(\frac{1}{2\alpha^2} - \frac{1}{\alpha}\right)\| x(t_k) \|^2 \\ &\quad + x(t_k)'\left(\frac{1}{\alpha}\xi_k - \frac{1}{\alpha^2}\xi_k + (1 - 1/\alpha)\xi_k^F T\right) \\ &\quad + \frac{1}{2}\left(\frac{1}{\alpha^2}\xi_k'\xi_k + \frac{2}{\alpha}\xi_k'\xi_k^F T + (\xi_k^F)'\xi_k^F T^2\right) \end{aligned}$$

Applying the requirement that $\xi_k \in \mathcal{X}$ (that is, $\| \xi_k \| \leq LT^q$) and $\xi_k^F \in \mathcal{XF}$ (that is, $\left\| \xi_k^F \right\| \leq LT^{q-1}$) allows us to find an upper bound on $\hat{V}(x(t_{k+1})) - V(x(t_k))$ as follows:

$$\begin{aligned} \hat{V}_{k+1} - V_k &\leq \frac{1}{2}\left(-\frac{2}{\alpha} + \frac{1}{\alpha^2}\right)\| x(t_k) \|^2 \\ &+ \left(1 + \frac{1}{\alpha}\right)^2 LT^q \| x(t_k) \| + \frac{1}{2}\left(1 + \frac{1}{\alpha}\right)^2 L^2T^{2q} \end{aligned}$$

So that (using $V = \frac{1}{2}x'x$):

$$\begin{aligned} \hat{V}_{k+1} \leq &(1 - 1/\alpha)^2 V_k + (1 + 1/\alpha)^2 LT^q \| x(t_k) \| \\ &+ \frac{1}{2}(1 + 1/\alpha)^2 L^2T^{2q} \end{aligned} \tag{20}$$

Since at each control time t_{k+j} (where $j < m$) we assume that the estimated value of the state is in fact exact, we can apply (20) recursively to get the following inequality:

$$\begin{aligned} \hat{V}_{k+m} \quad \leq \quad &(1 - 1/\alpha)^{2m} V_k \\ &+ \frac{1 - (1 - 1/\alpha)^{2m}}{2/\alpha - 1/\alpha^2}((1 + 1/\alpha)^2 LT^q \| x(t_k) \| \\ &+ \frac{1}{2}(1 + 1/\alpha)^2 L^2T^{2q}) \end{aligned}$$

From which we can see that $\hat{V}_{k+m} - V_k$ is bounded from above by:

$$\begin{aligned}&\hat{V}_{k+m} - V_k \\ &\leq -\frac{1}{2}(1-(1-1/\alpha)^{2m}) \parallel x(t_k) \parallel^2 \\ &+ \frac{1-(1-1/\alpha)^{2m}}{2/\alpha - 1/\alpha^2}((1+1/\alpha)^2 LT^q \parallel x(t_k) \parallel \\ &+ \frac{1}{2}(1+1/\alpha)^2 L^2 T^{2q})\end{aligned} \tag{21}$$

We compute an upper bound for $V_{k+m} - \hat{V}_{k+m}$ as follows:

$$\begin{aligned}V_{k+m} - \hat{V}_{k+m} =& \hat{x}(t_{k+m})'(x(t_{k+m}) - \hat{x}(t_{k+m})) \\ &+ \frac{1}{2} \parallel x(t_{k+m}) - \hat{x}(t_{k+m}) \parallel^2\end{aligned} \tag{22}$$

$$= \hat{x}(t_{k+m})'\xi_{k+m} + \frac{1}{2} \parallel \xi_{k+m} \parallel^2 \tag{23}$$

We next use the fact that $\hat{x}(t_{j+1}) = (I+AT)x(t_j) + AT\xi_j + T\xi_j^F$ to rewrite $x(t_{k+m})$ in terms of $x(t_k)$ (assuming that measurements are made at times t_k and t_{k+m}). First, note that

$$\hat{x}(t_{k+1}) = (I+AT)x_k + AT\xi_k + T\xi_k^F \tag{24}$$

Since no measurement is available at time t_{k+1}, we assume that $x(t_{k+1}) = \hat{x}(t_{k+1})$; therefore $\xi_{k+1} = 0$ and $\xi_{k+1}^F = 0$. Therefore, appying the above equation to $x(t_{k+1})$ we get:

$$\begin{aligned}&\hat{x}_{k+2} \\ =& \quad (I+AT)x(t_{k+1}) + AT\xi_{k+1} + T\xi_{k+1}F \\ =& \quad (I+AT)x(t_{k+1}) \\ =& \quad (I+AT)x(t_k) + (I+AT)AT\xi_k + (I+AT)T\xi_k^F\end{aligned}$$

Again, since there are no new measurements available, we define $x(t_{k+2}) = \hat{x}(t_{k+2})$. Applying the above equation recursively, we get:

$$\begin{aligned}\hat{x}(t_{k+m}) =& (I+AT)^m x(t_k) + (I+AT)^{m-1}AT\xi_k \\ &+ (I+AT)^{m-1}T\xi_k^F\end{aligned} \tag{25}$$

from which it follows that: ***correct the following to match changes made above (added a factor of AT in the ξ term in above equation for $\hat{x}(t_{k+m})$)*

$$\begin{aligned}&V_{k+m} - \hat{V}_{k+m} \\ =& (I+AT)^m x(t_k)'\xi_{k+m} + (I+AT)^{m-1}\xi_k'\xi_{k+m} \\ &+ (I+AT)^{m-1}T(\xi_k^F)'\xi_{k+m} + \frac{1}{2} \parallel \xi_{k+m} \parallel^2\end{aligned} \tag{26}$$

$$\begin{aligned}&\leq (1-1/\alpha)^m L(mT)^q \parallel x(t_k) \parallel \\ &+ (2(1-1/\alpha)^{m-1}m^q + \frac{1}{2}m^{2q})L^2T^{2q}\end{aligned} \tag{27}$$

Eq. (21) is a quadratic polynomial in $\parallel x(t_k) \parallel$ with negative leading-order coefficient, and (27) is an affine function in $\parallel x(t_k) \parallel$. Applying Lemma 3.1, we see that we can define h such that $V_{k+m} - V_k < 0$ whenever $\parallel x(t_k) \parallel > h$ where:

$$h = \frac{(b+d) + \sqrt{(b+d)^2 + 4a(c+f)}}{2a} \tag{28}$$

whith

$$a = \frac{1}{2}(1-(1-1/\alpha)^{2m}) \tag{29}$$

$$b = \frac{1-(1-1/\alpha)^{2m}}{2/\alpha - 1/\alpha^2}(1+1/\alpha)^2 LT^q \tag{30}$$

$$d = (1-1/\alpha)^m L(mT)^q \tag{31}$$

$$c = \frac{1-(1-1/\alpha)^{2m}}{2/\alpha - 1/\alpha^2}\frac{(1+1/\alpha)^2}{2}L^2T^{2q} \tag{32}$$

$$f = (2(1-1/\alpha)^{m-1}m^q + \frac{1}{2}m^{2q})L^2T^{2q} \tag{33}$$

■

The following theorem establishes the invariance of the neighborhood defined by $V \leq \frac{1}{2}h^2$:

Theorem 3.1: Consider the closed loop system dynamics given by (13)-(15), with control interval T and measurement interval mT. Assume that position error ξ belongs to $\mathcal{X}$ and velocity error ξ^F to $\mathcal{XF}$. The control gain is a constant, $\frac{1}{\alpha T}$. Then as $k \to \infty$, the asymptotic tracking error $\parallel x(t_k) \parallel$ is bounded above by h, which is defined as in eq. (28)-(33).

Proof: Consider the function W_k defined as $W_{k+m} = \max_{i \in (k,k+m]} V_i$. Consider a time t_k such that $W_k \geq \frac{1}{2}h^2$. Then, by Lemma 3.1, $V_{k+m} - V_k < 0$. Therefore, we will have $V_{k+m} < V_k$. Since increases in V_i occur only at measurement times, this also means that $W_{k+m} < W_k$ Therefore, as k increases, the value of W_k will decrease until $W_k \leq \frac{1}{2}h^2$.

Now consider $W_k < \frac{1}{2}h^2$. We will show that the value of W_{k+m} is bounded above by $\frac{1}{2}h^2$, so that once V_k is inside the set $\{V_k : V_k < \frac{1}{2}h^2\}$, it stays there for all time $t \geq t_k$. First, using the fact that $V_k = \frac{1}{2} \parallel x(t_k) \parallel^2$, we rewrite the sum of (21) and (27) as:

$$\begin{aligned}V_{k+m} \leq& \frac{1}{2}(1-1/\alpha)^{2m} \parallel x(t_k) \parallel^2 \\ &+ (b+d) \parallel x(t_k) \parallel + (c+f)\end{aligned} \tag{34}$$

$$= \tilde{a} \parallel x(t_k) \parallel^2 + \tilde{b} \parallel x(t_k) \parallel + \tilde{c} \tag{35}$$

where $\tilde{a}, \tilde{b}, \tilde{c} > 0$. The upper bound on V_{k+m} is a positive quadratic in $\parallel x(t_k) \parallel$ with value $\frac{1}{2}h^2$ at $x(t_k) = h$. By Lemma 3.2, $V_{k+m} \leq \frac{1}{2}h^2$ for all $\parallel x(t_k) \parallel \leq h$. Since increases in V_i (for $i \in (k, k+m]$) occur only at the measurement time t_{k+m}, $V_{k+m} \leq \frac{1}{2}h^2$ means that $V_i \leq \frac{1}{2}h^2$ for all $i \in (k, k+m]$, which implies that $W_{k+m} \leq \frac{1}{2}h^2$. Therefore W_{k+m} is bounded above by $\frac{1}{2}h^2$ for all $t \geq t_k$.

Therefore, as $k \to \infty$, $W_k \leq \frac{1}{2}h^2$, where h is given by (28)-(33). ■

The bound h on the steady-state tracking error is directly proportional to the control interval T and the measurement interval mT. In order to achieve the exact tracking i.e. $x(t_k) = 0$ for all k, no control or measurement delay should be tolerated. Therefore, under time delay, the best we can hope is to stabilize the target in a small neighborhood of the center of the FOV. The size of this neighborhood relates to the maximum possible delay through (28)-(33). If the delay is longer, the size of the neighborhood is bigger which implies that the tracking performance will be degraded.

IV. Tracking multiple targets with asynchronicity

Now consider the case where multiple targets are monitored by the camera $C1$, and one or more of them may be occluded. If occlusion does occur, camera $C1$ communicates with other cameras to obtain estimates for the states of the occluded targets. In order to reduce networking traffic, we assume that the communication between cameras happens infrequently, so that not all estimates made by these other cameras will be sent to the requesting camera $C1$. For example, if a camera has estimates available every 10ms, the camera may choose to send the estimates every 50ms. Therefore, at $C1$, estimates for hidden targets will be available less frequently than estimates for targets which have clear line of sight. We want to find out how frequently communication must happen to guarantee specified performance levels on target tracking.

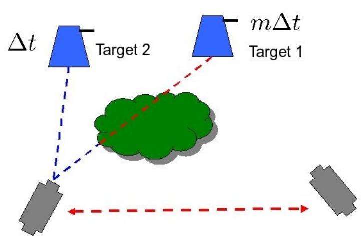

Fig. 2. Two-target tracking system with one occluded target

We analyze a simple case where we assume that there are two point targets ($N = 2$), of which one (say, target 1) is occluded (see fig. 2). Camera $C1$ generates updates for the postion of any unoccluded target with interval T. The estimate of the state of target 1 is communicated to $C1$ at time intervals of length mT, where integer $m > 1$. Suppose that these estimates are available at times $t_k, t_{k+m}, t_{k+2m}, ...$ On the interval $[t_k, t_{k+m})$, estimates for the unblocked target (target 2) are updated m times and the estimate for the occluded target (target 1) is updated only once. This causes asynchronicity in the discretized system

$$x_c(t_{k+1}) = x_c(t_k) + G_{c,k}u_kT + F_{c,k}T \tag{36}$$

where $x_c(t_k) = \frac{1}{N}\Sigma_{i=1}^N x_i(t_k)$, $G_{c,k} = G_c(t_k)$, and $F_{c,k} = \frac{1}{N}\Sigma_{i=1}^N B_i(t_k)$.

Consider the time interval $[t_k, t_{k+m})$. As in the single-target case, we use a simple linear predictor to construct position estimates for x_c:

$$\hat{x}_c(t_{j+1}) = x_c(t_j) + G_{c,j}\mathbf{u}_jT + F_{c,j}T \tag{37}$$

$$\hat{F}_{c,j+1} = \hat{F}_{c,j} \tag{38}$$

$$\mathbf{u}_{c,j} = \hat{G}_{c,j}^{-1}(A\hat{x}_c(t_j) - \hat{F}_{c,j}) \tag{39}$$

When no new estimate for target 1 is available, we assume that the estimated values for x_c and F_c are exact. As before, we can rewrite the state equations in the approximate form:

$$\hat{x}_c(t_{k+1}) = (I + AT)x_c(t_k) + \tilde{\mathbf{u}}_k \tag{40}$$

$$\tilde{\mathbf{u}}_k = AT\xi_{c,k} + \xi_{c,k}^F T \tag{41}$$

which is a perturbed linear system driven by stochastic noise inputs $\xi_{c,k}$ and $\xi_{c,k}^F$. This time, however, unlike the single-target case, each of the noise inputs has two distinct components, corrsponding to the errors on the states of targets 1 and 2: $\xi_{c,k} = \frac{1}{2}(\xi_{1,k} + \xi_{2,k})$ and $\xi_{c,k}^F = \frac{1}{2}(\xi_{1,k}^F + \xi_{2,k}^F)$. As in the single-target case, we assume that the error in estimated position for each of the targets is $\xi_{i,j} \in \mathcal{X} = \{\,\phi : \|\,\phi\,\| \le L(\Delta t)^q\,\}$, $\forall j$ and error in estimated velocity is $\xi_{i,j}^F \in \mathcal{XF} = \{\,\psi : \|\,\psi\,\| \le L(\Delta t)^{q-1}\,\}$, $\forall j$, $i = 1, 2$. Here $L \in \mathbb{R}^+$ and $q \in \mathbb{Z}^+$ are known, and Δt is the time since the last available measurement. The matrix A is selected to be $A = -K_kI$ with $K_k = 1/\alpha T$.

Lemma 4.1: Consider a camera tracking system with dynamics described by eq. (40)-(41). The inter-control input interval is T and the inter-measurement intervals are mT for target 1 and T for target 2. The matrix A and bounds on the estimates of states of the targets are defined as above. Then the system is discrete-time input to state stable (ISS) and admits a quadratic lyapunov function.

Proof: The updates to the state of target 1 and 2 are available with $\Delta t = mT$ and T, respectively. The maximum position error we may expect at any measurement time, therefore, is $\xi_c = \frac{1}{2}L((mT)^q + T^q)$; the maximum velocity error is $\xi_c^F = \frac{1}{2}L((mT)^{q-1} + T^{q-1})$. Both ξ_c and ξ_c^F are bounded for any given $T < \infty$.

With A defined as $A = -\frac{1}{\alpha T}I$, $I + AT$ is a Schur matrix. Therefore, as in the proof of Lemma 4.1, our system admits an ISS Lyapunov function, and is necessarily ISS. ■

Input to state stability of the system (40)-(41) is guaranteed by the above Lemma; the state will converge to some bounded neighbourhood of the origin, which can be estimated by constructing an explicit discrete ISS Lyapunov function V for the system and finding values of $x_c(t_k)$ such that $V_{k+m} - V_k < 0$. Estimating the size of the neighbourhood, however, is less straighforward than before.

Consider a simple quadratic ISS Lyapunov function for the system:

$$V_{c,k} = \frac{1}{2}x_c(t_k)^T x_c(t_k) \tag{42}$$

We will analyze the behavior of this function over a time interval $(t_k, t_{k+m}]$, where we assume that measurements of the state of target 1 are available at times t_k and t_{k+m}. Similar to the single-target case, $V_{c,k+m} - V_{c,k}$ can be as:

$$\begin{aligned} & V_{c,k+m} - V_{c,k} \\ &= V_{c,k+m} + \sum_{j=k+1}^{k+m-1}(-V_{c,j} + V_{c,j}) - V_{c,k} \end{aligned}$$

Thus, to guarantee $V_{c,k+m} - V_{c,k} < 0$, it is sufficient to have $V_{c,j+1} - V_{c,j} < 0$ for all $j \in \{\,k, k+1, ..., k+m-1\,\}$.

Lemma 4.2: For the system defined by (40)-(41) and discrete ISS Lyapunov function (42), , for all $\|\,x_c(t_j)\,\| > h_1$, where

$$h_1 = \frac{\alpha^2LT^q(1 + (1 + 1/\alpha)^2 + 2\sqrt{1 + 4/\alpha + 2/\alpha^2 + 1/\alpha^3})}{2(2\alpha - 1)}$$

we have $V_{c,j+1} - V_{c,j} < 0$, $\forall j \in \{\,k, k+1, ..., k+m-1\,\}$.

Proof: For all times $t \in \{ t_{k+1}, t_{k+2}, ..., t_{k+m-1} \}$, the error in the estimate of state of target 1 is 0. Therefore, at these times, the only error contribution in estimation of the state x_c is error in the estimate of the state of target 2, $\xi_{c,k} = \frac{1}{2}\xi_{2,k}$ and $\xi^F_{c,k} = \frac{1}{2}\xi^F_{2,k}$, where $\xi_2 \in \mathcal{X}$ and $\xi^F_2 \in \mathcal{XF}$. With this in mind we write $V_{c,j+1} - V_{c,j}$ ($j \in \{ k, k+1, ..., k+m-1 \}$) as:

$$V_{c,j+1} - V_{c,j} = V_{c,j+1} - \hat{V}_{c,j+1} + \hat{V}_{c,j+1} - V_{c,j} \tag{43}$$

We analyze $V_{c,j+1} - \hat{V}_{c,j+1}$ and $\hat{V}_{c,j+1} - V_{c,j}$ separately.

First, note that since the error bound and control law are identical to the single-target case, $\hat{V}_{c,j+1} - V_{c,j}$ is bounded just as in the single-target case:

$$\begin{aligned}&\hat{V}_{c,j+1} - V_{c,j}\\ &\leq -\frac{1}{2}\left(\frac{2}{\alpha} - \frac{1}{\alpha^2}\right) \| x_c(t_k) \|^2\\ &+ \frac{1}{2}\left(1 + \frac{1}{\alpha}\right)^2 LT^q \| x_c(t_k) \| + \frac{1}{8}\left(1 + \frac{1}{\alpha}\right)^2 L^2T^{2q}\end{aligned} \tag{44}$$

Similarly, $V_{c,j+1} - \hat{V}_{c,j+1}$ can be written as:

$$\begin{aligned}&V_{c,j+1} - \hat{V}_{c,j+1}\\ &= (x_c(t_{j+1}) + \xi_{c,j+1})'\xi_{c,j+1} + \frac{1}{2} \| \xi_{c,j+1} \|^2\end{aligned} \tag{45}$$

Which is bounded above as follows:

$$V_{c,j+1} - \hat{V}_{c,j+1} \leq \frac{1}{2}LT^q \| x_c(t_{j+1}) \| + \frac{3}{8}L^2T^{2q} \tag{46}$$

Applying Lemma 3.1 to (44) and (46), we see that $V_{c,j+1} - V_{c,j} < 0$ whenever $\| x(t_j) \| > h_1$, where h_1 is given by:

$$h_1 = \frac{\alpha^2 LT^q(1 + (1 + 1/\alpha)^2 + 2\sqrt{1 + 4/\alpha + 2/\alpha^2 + 1/\alpha^3})}{2(2\alpha - 1)} \tag{47}$$

■

Lemma 4.3: For the system defined by (40)-(41) and discrete ISS Lyapunov function (42),

$$\max\{ V_{c,k+1}, ..., V_{c,k+m-1} \} \leq \max\{ V_{c,k}, \frac{1}{2}h_1^2 \}$$

where h_1 is defined as in (47).

Proof: First suppose that $x_c(t_k) > h_1$, corresponding to $V_{c,k} > \frac{1}{2}h_1^2$. By Lemma 4.2, this means that $V_{c,k+1} - V_{c,k} < 0$, so that $V_{c,k+1} < V_{c,k}$. By induction, $V_{c,j+1} < V_{c,j} < V_{c,k}$ holds for each $j \in \{ k+1, ..., k+m-1 \}$ so long as $x_c(t_j) > h_1$.

Now suppose that for some j, $\| x_c(t_k) \| \leq h_1$. Then, adding equations (44) and (46) for time t_j, we get:

$$\begin{aligned}&V_{c,j+1} - V_{c,j}\\ &\leq -\frac{1}{2}\left(\frac{2}{\alpha} - \frac{1}{\alpha^2}\right) \| x_c(t_k) \|^2\\ &+ \left(\frac{1}{\alpha} + 1\right) LT^q \| x_c(t_k) \| + \frac{1}{2}L^2T^{2q}\end{aligned} \tag{48}$$

Adding $V_{c,j} = \frac{1}{2} \| x_c(t_j) \|$ to both sides of the inequality, we see that:

$$\begin{aligned}&V_{c,j+1}\\ &\leq \frac{1}{2}\left(1 - \frac{1}{\alpha}\right)^2 \| x_c(t_j) \|^2\\ &+ \left(\frac{1}{\alpha} + 1\right) LT^q \| x_c(t_j) \| + \frac{1}{2}L^2T^{2q}\\ &= \tilde{a} \| x_c(t_j) \|^2 + \tilde{b} \| x_c(t_j) \| + \tilde{c}\end{aligned} \tag{49}$$

where $\tilde{a}, \tilde{b}, \tilde{c}$ are striclty greater than 0 and $\tilde{a}h_1^2 + \tilde{b}h_1 + \tilde{c} = \frac{1}{2}h_1^2$. By Lemma 3.2, therefore, $V_{c,j+1} < \frac{1}{2}h_1^2$ for all $\| x_c(t_j) \| < h_1$.

Overall, for all initial values $V_{c,k}$,

$$\max\{ V_{c,k+1}, ..., V_{c,k+m} \} \leq \max\{ V_{c,k}, \frac{1}{2}h_1^2 \}$$

. ■

Lemma 4.4: For the system defined by (40)-(41) and discrete ISS Lyapunov function (42), there exists an $h_2 > h_1$ such that $V_{c,k+m} - V_{c,k+m-1} < 0$ for all $\| x_c(t_{k+m-1}) \| > h_2$.

Proof: At time t_{k+m}, estimates of the states of both targets are available, so that the error in position is given by $\xi_{c,k+m} = \frac{1}{2}(\xi_{1,k+m} + \xi_{2,k+m})$, where $\xi_{1,k+m} \in \mathcal{X}$ with $\Delta t = mT$ and $\xi_{2,k+m} \in \mathcal{X}$ with $\Delta t = T$. The error in velocity is given by $\xi^F_{c,k+m} = \frac{1}{2}(\xi^F_{1,k+m} + \xi^F_{2,k+m})$, where $\xi^F_{1,k+m} \in \mathcal{XF}$ with $\Delta t = mT$ and $\xi^F_{2,k+m} \in \mathcal{XF}$ with $\Delta t = T$. Thus, we can say that $\xi_{c,k+m} \in \{ \phi : \| \phi \| \leq \frac{1}{2}(1 + m^q)LT^q \}$ and $\xi^F_{c,k+m} \in \{ \psi : \| \psi \| \leq \frac{1}{2}(1 + m^{q-1})LT^{q-1} \}$.

We write $V_{c,k+m} - V_{c,k+m-1}$ as:

$$\begin{aligned}&V_{c,k+m} - V_{c,k+m-1}\\ &= V_{c,k+m} - \hat{V}_{c,k+m} + \hat{V}_{c,k+m} - V_{c,k+m-1}\end{aligned} \tag{50}$$

and analyze $V_{c,k+m} - \hat{V}_{c,k+m}$ and $\hat{V}_{c,k+m} - V_{c,k+m-1}$ separately.

It can be shown that:

$$\begin{aligned}&\hat{V}_{c,k+m} - V_{c,k+m-1}\\ &\leq -\frac{1}{2}\left(\frac{2}{\alpha} - \frac{1}{\alpha^2}\right) \| x_c(t_{k+m-1}) \|^2\\ &+ \frac{1}{2}\left(1 + \frac{1}{\alpha}\right)^2 LT^q \| x_c(t_{k+m-1}) \| + \frac{1}{8}\left(1 + \frac{1}{\alpha}\right)^2 L^2T^{2q}\end{aligned} \tag{51}$$

Then, we can find a bound on $V_{c,k+m} - \hat{V}_{c,k+m}$ as:

$$\begin{aligned}&V_{c,k+m} - \hat{V}_{c,k+m}\\ &\leq \frac{1}{2}\left(1 + m^q\right) LT^q \| x_c(t_{k+m}) \| + \frac{3}{8}\left(1 + m^q\right) L^2T^{2q}\end{aligned} \tag{52}$$

Adding together equations (51) and (52) and applying Lemma 3.1, we see that $V_{c,k+m} - V_{c,k+m-1} < 0$ whenever $\| x(t_j) \| > h_2$, where h_2 is given by:

$$h_2 = \frac{\alpha^2 LT^q(1 + (1 + 1/\alpha)^2 + 2\sqrt{H})}{2(2\alpha - 1)} \tag{53}$$

where

$$H = 1+4/\alpha+2/\alpha^2+1/\alpha^3+m^{2q}+(4+\frac{6}{\alpha}+\frac{1}{\alpha^2})m^q \quad (54)$$

Note that $h_2 > h_1$. ■

Lemma 4.5: For the system (40)-(41) and discrete ISS Lyapunov function (42),

$$\max\{ V_{c,k+1}, ..., V_{c,k+m} \} \leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \}$$

where h_2 is defined as in (53).

Proof: By Lemma 4.3, we already know that

$$\max\{ V_{c,k+1}, ..., V_{c,k+m-1} \} \leq \max\{ V_{c,k}, \frac{1}{2}h_1^2 \}$$

where $h_1 < h_2$, so that $\frac{1}{2}h_1^2 < \frac{1}{2}h_2^2$.

Suppose that $V_{c,k+m-1} > \frac{1}{2}h_2^2$. This means that $V_{c,k+m-1} > \frac{1}{2}h_1^2$, and therefore, perforce, $V_{c,k+m-1} < V_k$. In this case, by Lemma 4.4, $V_{c,k+m} - V_{c,k+m-1} < 0$, so that $V_{c,k+m} < V_{c,k+m-1}$, and we have established that

$$\max\{ V_{c,k+1}, ..., V_{c,k+m-1} \} = V_{c,k} \leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \}$$

Now suppose that $V_{c,k+m-1} \leq \frac{1}{2}h_2^2$. Adding equations (51) and (52) for time t_{k+m-1}, we get:

$$\begin{aligned} &V_{c,k+m} - V_{c,k+m-1} \\ &\leq -\frac{1}{2}\left(\frac{2}{\alpha} - \frac{1}{\alpha^2}\right) \| \, x_c(t_k) \, \|^2 \\ &+ \frac{1}{2}\left((1+1/\alpha)^2 + 1 + m^q\right) LT^q \, \| \, x_c(t_k) \, \| \\ &+ \frac{1}{8}((1+1/\alpha)^2 + 3(1+m^q))L^2T^{2q} \end{aligned} \quad (55)$$

Then, adding $V_{c,m+k-1} = \frac{1}{2} \| \, x_c(t_{m+k-1}) \, \|$ to both sides of the inequality, we see that:

$$\begin{aligned} &V_{c,k+m} \\ &\leq \frac{1}{2}\left(1 - \frac{1}{\alpha}\right)^2 \| \, x_c(t_{k+m-1}) \, \|^2 \\ &+ \left(\frac{1}{\alpha}+1\right) LT^q \, \| \, x_c(t_{k+m-1}) \, \| + \frac{1}{2}L^2T^{2q} \\ &= \tilde{a} \, \| \, x_c(t_{k+m-1}) \, \|^2 + \tilde{b} \, \| \, x_c(t_{k+m-1}) \, \| + \tilde{c} \end{aligned} \quad (56)$$

where $\tilde{a}, \tilde{b}, \tilde{c}$ are stricly greater than 0, and $\tilde{a}h_2^2 + \tilde{b}h_2 + \tilde{c} = \frac{1}{2}h_2^2$. Therefore, by Lemma 3.2, $V_{c,k+m} < \frac{1}{2}h_2^2 \leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \}$ for all $\| \, x_c(t_{k+m-1}) \, \| \leq h_2$.

Since it is known that

$$\begin{aligned} \max\{ V_{c,k+1}, ..., V_{c,k+m-1} \} &\leq \max\{ V_{c,k}, \frac{1}{2}h_1^2 \} \\ &\leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \} \end{aligned} \quad (57)$$

And we have shown that for arbitrary $V_{c,k+m-1}$, $V_{c,k+m} \leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \}$, it follows that

$$\max\{ V_{c,k+1}, ..., V_{c,k+m} \} \leq \max\{ V_{c,k}, \frac{1}{2}h_2^2 \}$$

■

Lemma (4.5) implies that as $k \to \infty$, $x_c(t_k)$ converges to a neighbourhood of size h_2 around the center of C1's FOV. This upper bound on the size of the neighbourhood will tend to be conservative, as we have always assumed a worst-case estimation error for the state of each of the targets. However, it is a useful metric for estimating the performance of the networked camera tracking system given computation and communication delay and synchronicity.

V. Simulation

The above camera tracking system is simulated in Matlab for both the single-target and the two-target asynchronous measurement case. Arbitrary deterministic 2-dimensional trajectories are assigned to the targets. It was assumed that the camera system can observe the location of each object with additive Gaussian white noise.

Given the nonlinear dynamics of the targets and measurement noise, it is desirable to use a short-memory filter to estimate the state x from a given observation. A polynomial predictor (implemented using Matlab `polyfit` function) was used to estimate the state at each control instant. The ten most recent data points were used to find a least-squares second-order polynomial function for x_1 and x_2 over time.

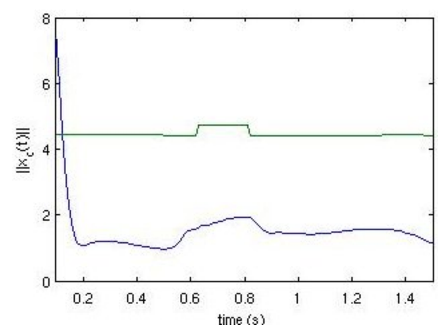

Fig. 3. Distance of target to center of C1's FOV (blue), and theoretical bound (green). The changes bound is higher when one or both of the targets are occluded.

The simulation was run with $T = 10^{-3}$ sec and m varying between 1 (target not occluded) to 5 (target occluded). A section of the target space was designated "brush area", with targets becoming occluded whenever they entered this area. As illustrated in fig. 3, the estimates of the size of the neighborhood of the center of the camera's FOV in the previous sections tend to be much higher than the neighborhoods obtained in simulation. This is not surprising, since we have always assumed a worst-case scenario, and calculated bounds on $V(x(t))$ corresponding to maximum possible error at every measurement instant. It may be observed, however, that the size of the neighborhood to which x converges is dependent on mT, as expected from our model and calculations.

References

[1] S. Velipasalar, J. Schlessman, C.-Y. Chen, W. Wolf, and J. P. Singh, "SCCS: a scalable clustered camera system for multiple object tracking communicating via message passing interface," in *Proc. International Conference on Multimedia and Expo*, 2006.

[2] L. Dugard and E. I. Verriest, *Stability and control of time-delay systems*, ser. Lecture notes in control and information sciences ; 228. London ; New York: Springer, 1998.

[3] M. S. Mahmoud, *Robust control and filtering for time-delay systems*. New York: Marcel Dekker, 2000.
[4] J. K. Hale, *Functional differential equations*. minus 0.4emNew York,: Springer-Verlag, 1971.
[5] M. Malek-Zavarei and M. Jamshidi, *Time-delay systems : analysis, optimization, and applications*. New York, N.Y., U.S.A.: Elsevier, 1987.
[6] M. Cao, A. Morse, and B. Anderson, "Agreeing asynchronously in continuous time," *preprint*, 2006.
[7] J. N. Tsitsiklis and D. Bertsekas, "Distributed asynchronous routing in data communication networks," *IEEE Transactions on Automatic Control*, vol. 31, pp. 325–332, 1986.
[8] A. Balluchi, P. Murrieri, and A. L. Sangiovanni-Vincentelli, "Controller synthesis on non-uniform and uncertain discrete-time domains," in *HSCC 2005, LNCS 3414*, M. Morari and L. Thiele, Eds. Springer-Verlag, 2005, pp. 118–133.
[9] J. Grizzle and J.-M. Kang, "Discrete-time control design with positive semi-definite Lyapunov functions," *Systems and Control Letters*, vol. 40, no. 3, pp. 329–342, 1984.
[10] T. Hu and Z. Lin, "Absolute stability analysis of discrete-time systems with composite quadratic Lyapunov functions," *IEEE Transactions on Automatic Control*, vol. 50, no. 6, pp. 781–797, 2005.
[11] C. Macnab and G. D'Eleuterio, "Discrete-time Lyapunov design for neuroadaptive control of elastic-joint robots," *International Journal of Robotic Research*, vol. 19, no. 5, pp. 511–525, 2000.
[12] K. M. Passino, A. N. Michel, and P. J. Antsaklis, "Lyapunov stability of a class of discrete event systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 2, pp. 269–279, 1994.
[13] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Trans. Automat. Cont.*, vol. 43, pp. 475–482, 1998.
[14] Z.-P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica*, vol. 37, pp. 857–869, 2001.
[15] E. Sontag, "Further facts about input to state stabilization," *IEEE Trans. Automatic Control*, vol. 35, no. 4, pp. 435–443, 1990.
[16] ——, "Smooth stabilization implies coprime factorization," *IEEE Trans. Automatic Control*, vol. 34, no. 4, pp. 473–476, 1989.
[17] D. Nesic and A. R. Teel, "Input-to-state stability of networked control systems," *Automatica*, vol. 40, no. 12, pp. 2121–2128, 2004.
[18] W. P. Dayawansa, B. K. Ghosh, C. Martin, and W. Xiaochang, "A necessary and sufficient condition for the perspective observability problem," *Systems and Control Letters*, vol. 25, no. 3, pp. 159–66, 1995.
[19] B. K. Ghosh and E. P. Loucks, "A perspective theory for motion and shape estimation in machine vision," *SIAM Journal on Control and Optimization*, vol. 33, no. 5, pp. 1530–59, 1995.
[20] L. Ma, C. Cao, N. Hovakimyan, W. E. Dixon, and C. Woolsey, "Range identification in the presence of unknown motion parameters for perspective vision systems," in *Proc. 2007 American Control Conference*, New York, 2007, pp. 972–977.