Diagnosis and Prognosis of Scrubber Faults for Underwater Rebreathers based on Stochastic Event Models

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Abstract—Improper CO₂ removal mechanisms of CO₂ scrubbers often lead to the existence of CO₂ in gas inhaled by a diver from underwater rebreathers. This may cause CO₂ related rebreather faults and subsequently would increase the risk of human injuries. We introduce a stochastic model for three CO₂ related rebreather faults: CO₂ bypass, scrubber exhaustion, and scrubber breakthrough. We establish the concept of CO₂ channeling that describes the cause of the faults and present a CO₂ channeling model based on a stochastic process driven by a Poisson counter. This helps us to investigate how CO₂ flow inside the rebreather is affected by CO₂ related faults. Fault diagnosis/prognosis algorithms are developed based on the stochastic model and are tested in simulation.

I. INTRODUCTION

Underwater breathing apparatus (UBA) are used for dive assistance. In consideration of military use or saturation diving, one of the most widely used UBA is the closed circuit rebreather (CCR) [1]. A rebreather reuses exhaled gas from divers by employing a scrubber to remove CO₂ from the exhaled gas. However, CO₂ removal mechanisms are usually not perfect. The phenomena of excessive CO₂ in inhaled gas and the loss of CO₂ absorption capability of the scrubber are considered as severe faults of the rebreather, which may lead to life threatening injuries to divers due to the intake of the excessive CO₂ [2, 3, 4].

Efforts to analyze or detect possible faults of rebreathers have been made in [4], [5], [6]. Deep Life, Ltd. published research data on rebreathers, and their work including a MATLAB/SIMULINK rebreather model is accessible [4]. A computer simulation model for the chemical kinetics of CO₂ absorption by a scrubber was developed in [5] to investigate the characteristics of the scrubber. This model is utilized to monitor the status of the scrubber by observing temperature changes induced by chemical reactions between CO₂ and the scrubbing chemicals. In addition, a fault tree was designed in [6] to identify risks of rebreather faults.

Aside from rebreather fault modeling, the physical characteristics of UBA such as gas dynamics under pressure at various locations are modeled in [7], [8], [9]. A computer simulation tool [7] analyzes the overall gas flow in the UBA. The breathing dynamics of closed-circuit UBA or CCRs are discussed in [8]. The main interest there is how to maximize performance in terms of breathing characteristics including work of breathing and peak to peak pressure. The authors of [9] extended [7], [8] by applying network theory to describe components in the breathing loop that affect gas flow.

In this paper, we focus on CO₂ related faults of a rebreather and model how CO₂ flow inside the rebreather is affected by CO₂ related faults. The faults we are interested in include CO₂ bypass, scrubber exhaustion, and scrubber breakthrough. We define a random event that a small amount of CO₂ passes through the scrubber without being absorbed as CO₂ channeling, which can be described by a Poisson process. Accumulated CO₂ channeling will lead to the faults. The CO₂ flow will be affected by the faults and can be modeled using stochastic differential equations.

Our approach on the stochastic representation of CO₂ related rebreather faults is novel. The model captures the stochastic nature of faults that are difficult to detect and predict. This leads to the development of stochastic model based fault diagnosis/prognosis algorithms for CO₂ related rebreather faults without modeling the process of chemical reactions inside a scrubber which might require more computing power. Our contribution may lead to detection and prediction of scrubber faults in real time.

In the next section, we briefly explain terminologies and mechanisms of rebreathers, and in Section III, we model respiration of divers that provides the input and the output of a rebreather. In Section IV, we simplify gas dynamics of an oxygen rebreather to focus on CO₂ related scrubber faults and investigate the influence of the faults on CO₂ flow in the simplified rebreather by introducing a CO₂ channeling model driven by a stochastic process. The description of the particle filter for the CO₂ flow model is presented in Section V followed by the introduction to fault diagnosis and prognosis for CO₂ related rebreather faults in Section VI. Simulation results are shown in Section VII and conclusions are provided.
II. MECHANISMS OF A REBREATHER

As shown in Fig. 1, a typical oxygen rebreather consists of a mouth-piece, a scrubber canister, an oxygen cylinder and a pressure gauge. Air flow inside a rebreather is driven by a diver’s exhalation and inhalation forming a closed breathing loop [10]. From the viewpoint of the scrubber, the portion of the breathing loop where gas flows from the diver through the mouth-piece to the scrubber is named the incoming path, and the portion of the breathing loop where gas flows out of the scrubber back to the diver is named the outgoing path. The direction of gas flow is regulated by one-way valves in the mouth-piece. When a diver inhales, the valves open the incoming path and direct the exhaled gas to the incoming path to prevent it from being mixed with the gas in the outgoing path. During the use of the rebreather, pure oxygen is injected into the breathing loop from the oxygen cylinder. This can be performed either actively at a constant rate, which must satisfy the individual diver’s personal needs based on the diver’s metabolism, or by manually adding oxygen using a hand-operated valve. It can also be controlled based on the difference between ambient pressure and the breathing loop pressure. The pressure of the oxygen inside the cylinder is checked with the pressure gauge.

![Fig. 1. A pure oxygen rebreather (Cobra).](image)

CO₂ is absorbed by the scrubber using chemicals such as soda-lime stored in the canister [11], [12], [13]. The scrubber has a limited life time since the reaction of the scrubbing chemicals with CO₂ is irreversible. In fact, even when soda-lime is not fully consumed, CO₂ may pass through the scrubber by a large amount, causing CO₂ related faults. Thus, the status of the scrubber consumption should be monitored in order for divers to avoid severe injuries due to the intake of excessive CO₂, i.e., hypercapnia. However, due to their stochastic nature, these faults are difficult to predict, and when they happen, divers often have little time to react. One of our goals is to understand the stochastic nature of the CO₂ related faults towards better predictions.

III. MODELING INHALATION AND EXHALATION RATES

Divers interact with rebreathers by respiration, i.e., inhalation and exhalation. A rebreather can be viewed as a system that takes exhalation as an input and produces an output for inhalation. An advanced computer simulation model of the human respiration is dealt with in [14] and its application to semi-closed circuit underwater breathing equipment is discussed in [15]. For rebreather fault simulations, a simplistic respiration model is constructed in this section in accordance with European Standard EN14143:2003 [16].

European standard EN14143:2003 [16] requires a sinusoidal waveform of breathing simulator and recommends CO₂ absorption endurance test at ventilation rate 40 L/min and at CO₂ generation rate 1.6 L/min. Moreover, the breathing simulator should handle breathing frequency of 20/min when ventilation rate is 40 L/min. According to these recommendations, we define the ventilation rate \( r \) and the frequency \( f \) as \( r = 40 [L/min] \) and \( f = \frac{1}{2} [Hz] \). Note that the period \( T \) is just \( \frac{1}{f} \) such that \( T = 3 [s] \).

In reality, respiration rates are different from person to person even among people who have similar physical conditions. We can easily see that the respiration rate is expected to vary along with workload changes which have influence on the heart rate. Let us define \( s = \frac{HR}{HR_{ref}} \) as the heart rate ratio where \( HR_{ref} \) is the reference heart rate and \( HR \) is the present heart rate. Let \( \xi(s) \) be a function which affects the magnitude of the respiration rate and \( \chi(t) \) be a function which affects the frequency of the respiration rate. Based on the above discussion, the normalizing factor \( L \), the inhalation rate \( v \) and the exhalation rate \( u \) with heart rate variations are

\[
L(s,t) = \begin{cases} \frac{(n+\frac{1}{2})T}{\xi(\chi(t))} \sin(2\pi f \chi(t) t)dt, & t \in I_1 \\ \frac{\xi(\chi(t))}{(n+1)T} \sin(2\pi f \chi(t) (t+\pi))dt, & t \in I_2 \end{cases}
\]

\[
v(s,t) = \begin{cases} \frac{r \xi(s)}{60fL(s,t)} \sin(2\pi f \chi(t) t), & t \in I_1 \\ 0, & t \in I_2 \end{cases}
\]

\[
u(s,t) = \begin{cases} 0, & t \in I_1 \\ \frac{r \xi(s)}{60fL(s,t)} \sin(2\pi f \chi(t) (t+\pi)), & t \in I_2 \end{cases}
\]

where \( \xi(1) = 1 \), \( \chi(1) = 1 \), \( I_1 = [\frac{nt}{2\chi(1)}; \frac{(n+\frac{1}{2})T}{\chi(1)}] \) and \( I_2 = [\frac{(n+\frac{1}{2})T}{\chi(1)}; \frac{(n+1)T}{\chi(1)}] \) for \( n = \{0,1,2,\cdots\} \).

The inhalation rate and the exhalation rate are listed separately as in (2) and (3) since inhalation and exhalation do not happen at the same time. A combination of one cycle of
inhalation and one cycle of exhalation produces a single cycle of respiration. In other words, in the case that the heart rate ratio \( s \) is one and the respiration rate is \( 1/3 \) \( HZ \), then each of inhalation and exhalation takes place in every 1.5 seconds, leading to a cycle of respiration in every 3 seconds.

During the inhalation cycle, the volume of inhaled \( CO_2 \) is determined by the inhalation rate \( v \) and the ratio of \( CO_2 \) in the outgoing path \( c_1 \). Let us define \( y \) as the volume of \( CO_2 \) removed by a scrubber. By our assumptions, gas in the incoming path does not flow to the outgoing path during inhalation, so \( CO_2 \) absorption \( y \) is zero. On the other hand, the amount of \( CO_2 \) in the outgoing path \( x \) will be reduced due to the inhalation by a diver. Therefore, \( CO_2 \) flow during inhalation \( (t \in I_1) \) is described as

\[
\dot{x} = -c_1(t) \cdot v(t) = -\frac{v(t)}{V_L} \cdot x(t) \tag{5}
\]

\[
y = 0 \tag{6}
\]

where \( x(0) = 0 \), \( y(0) = 0 \) and \( I_1 = \left[ \frac{nT}{\chi^{(s)}}, \frac{(n+1)T}{\chi^{(s)}} \right] \) for \( n = \{0,1,2,\cdots\} \).

We now consider the exhalation cycle. According to [17], the volume of \( CO_2 \) generated by a diver changes along with the heart rate variations. However, the variations of the \( CO_2 \) generation rate, which is related to \( O_2 \) metabolic consumption rate, based on workload is not easy to predict [10]. We use a simplified model based on the heart rate ratio here. We assume that the change in \( CO_2 \) generation rate is proportional to the heart rate ratio \( s \). Let us define the \( CO_2 \) generation ratio \( c_2 \) as

\[
c_2(s) = a \cdot s \tag{7}
\]

where \( a \) is a scaling factor with a typical value \( a = 0.04 \) and \( s \) is the heart rate ratio.

In the ideal cases, the scrubber absorbs all the \( CO_2 \) coming into a scrubber during exhalation. In other words, no \( CO_2 \) will flow to the outgoing path of a rebreather. The volume of \( CO_2 \) coming into a scrubber is determined by the exhalation rate \( u \) and the \( CO_2 \) generation ratio \( c_2 \). Thus, \( x \) and \( y \) during exhalation \( (t \in I_2) \) can be described as follows.

\[
\dot{x} = 0 \tag{8}
\]

\[
\dot{y} = c_2(s) \cdot u(t) \tag{9}
\]

where \( x(0) = 0 \), \( y(0) = 0 \), \( I_2 = \left[ \frac{(n+1/2)T}{\chi^{(s)}}, \frac{(n+1)T}{\chi^{(s)}} \right] \) for \( n = \{0,1,2,\cdots\} \) and \( s \) is the heart rate ratio. When \( CO_2 \) related faults happen, \( CO_2 \) flow will change as will be discussed in the following subsections.

### B. Influence of \( CO_2 \) related Faults on Respiration

Since \( CO_2 \) related faults cause \( CO_2 \) leaks to the outgoing path from the incoming path without being absorbed by a scrubber, the volume of \( CO_2 \) in the outgoing path \( x \) during exhalation is not zero anymore. Moreover, divers inhale \( CO_2 \) and this affects the volume of \( CO_2 \) contained in exhaled breath. Assuming that human-beings do not consume \( CO_2 \), then \( CO_2 \) breathed in will remain intact in exhaled gas. Let us define \( \delta c \) describing the total amount of \( CO_2 \) inhaled by a diver over one inhalation period as
\[\delta x(n) = x(nT) - x((n + \frac{1}{2})T) \quad (10)\]

where \(n = \{0, 1, 2, \cdots\}\). Assuming that all the CO\(_2\) inhaled by a diver is exhaled during the next exhalation cycle without loss, we define an additional exhalation rate \(u_1\) for this portion of returned CO\(_2\) as follows.

\[
u_1(t, n) = \begin{cases} 0 & , t \in I_1 \\ \frac{\delta x(n)}{T} \sin(2\pi ft + \pi) & , t \in I_2 \\
\end{cases} \quad (11)\]

where \(I_1 = \left[ \frac{nT}{\chi(s)} + \frac{(n+\frac{1}{2})T}{\chi(s)} \right]\) and \(I_2 = \left[ \frac{(n+\frac{1}{2})T}{\chi(s)} + \frac{(n+1)T}{\chi(s)} \right]\) for \(n = \{0, 1, 2, \cdots\}\).

Considering that the returned CO\(_2\) is not involved in CO\(_2\) generation, we define the volume of CO\(_2\) in exhaled breath using \(u\) and \(u_1\) as

\[
u(t, n) = \begin{cases} 0 & , t \in I_1 \\ c_2(s)(u(t) - u_1(t, n)) + u_1(t, n) & , t \in I_2 \\
\end{cases} \quad (12)\]

where \(s\) is the heart rate ratio.

### C. CO\(_2\) Channeling and Stochastic Modeling

We focus on the following CO\(_2\) related rebreather faults: CO\(_2\) bypass, scrubber exhaustion, and scrubber breakthrough. The descriptions of the faults are as follows. (i) CO\(_2\) bypass: Existence of CO\(_2\) beyond a safety level in the outgoing path of the rebreather. (ii) Scrubber exhaustion: Complete consumption of the CO\(_2\) scrubber. (iii) Scrubber breakthrough: Loss of the CO\(_2\) absorption capability of the CO\(_2\) scrubber before the depletion of the scrubber. We will see later that these faults can be rigorously defined using a stochastic model.

Typically, even when a scrubber functions properly, a small amount of CO\(_2\) will pass through the scrubber from the incoming path to the outgoing path. We define this phenomenon as CO\(_2\) channeling. Each CO\(_2\) channeling event happens randomly and is not considered as a fault. However, CO\(_2\) channeling will lead to the three CO\(_2\) related faults, and our stochastic model will reveal the mechanism.

Considering the randomness of CO\(_2\) channeling events, an arrival of CO\(_2\) at the outlet of a scrubber canister can be modeled as a Poisson process. We define a Poisson counter \(N(t)\) such that \(N(t)\) satisfies the following Poisson distribution:

\[
P[N(t+\tau) - N(t) = d] = \frac{e^{-\lambda\tau}(\lambda\tau)^d}{d!} \quad (13)\]

where \(d = \{0, 1, \cdots\}\) is the index of events and \(\lambda\) is the expected number of events over unit length of time. Since each CO\(_2\) channeling event happens independently, we define \(dN(t)\) as one single CO\(_2\) channeling event over \(dt\).

The amount of the CO\(_2\) channeling during one event can be modeled by a scaling factor, and it is affected by two factors: the total volume of CO\(_2\) absorbed by the scrubber \(y\) and the volume of CO\(_2\) coming into a scrubber canister \(\nu\). Then, the scaling factor \(G\) can be described as a function of \(y\) and \(\nu\) such that

\[G = G(y(t), \nu(t, n)). \quad (14)\]

Since we define \(dN(t)\) as an event of CO\(_2\) channeling within \(dt\), the change in CO\(_2\) volume in the outgoing path due to CO\(_2\) channelings can be expressed as

\[dx = G(y(t), \nu(t, n))dN(t). \quad (15)\]

The two parameters for the Poisson process, \(G\) and \(\lambda\), need to be carefully selected to reflect the physical behavior of the rebreather. At the beginning of the rebreather use, CO\(_2\) channelings are very unlikely, but as the scrubber being consumed, the probability of CO\(_2\) channeling increases. After the scrubber is depleted, CO\(_2\) channeling will happen with probability \(1\). According to this, \(\lambda\) of the Poisson process can be assumed to be zero for a new scrubber but will become \(\infty\) as the scrubber is getting fully consumed. Suppose the amount of CO\(_2\) absorbed by the scrubber \(y\) is zero initially and its maximum capacity is \(y_{\text{max}}\), then the rate parameter should satisfy

\[
\lambda(y) = \begin{cases} 0 & , y = 0 \\ \frac{y(t)}{y_{\text{max}} - y(t)} & , y > 0 \\
\end{cases} \quad (18)\]

where \(y_{\text{max}}\) is the maximum capacity of the CO\(_2\) scrubber and \(y(t)\) is the remaining CO\(_2\) absorption capacity of the CO\(_2\) scrubber.

The function \(G(y, \nu)\) will describe how much of CO\(_2\) could pass through a scrubber when a CO\(_2\) channeling event happens. Initial possible amount of CO\(_2\) channeling can be assumed to be zero for a new scrubber, but when a CO\(_2\) scrubber reaches its maximum capacity, all CO\(_2\) coming into the scrubber will pass through. Let us assume that \(G\) is a polynomial function increasing from zero to \(\nu\) as \(y(t)\) goes from zero to \(y_{\text{max}}\), then one possible choice of function \(G\) is

\[G(y, \nu) = \left(\frac{y(t)}{y_{\text{max}}}ight)^\alpha \nu(t, n) \quad (19)\]

where \(\alpha > 0\) is a stretching factor. We see that when \(y = 0\), no CO\(_2\) channeling can happen, but when \(y = y_{\text{max}}\), all CO\(_2\) in the incoming path will pass through the scrubber in a single channeling event.

### D. CO\(_2\) Flow under CO\(_2\) related Faults

In this subsection, we will specify CO\(_2\) flow dynamics in the rebreather induced by CO\(_2\) related faults using the stochastic CO\(_2\) channeling model with respect to \(x\) and \(y\). Parameters and variables of the model are summarized in table I.

By the assumption that gas in the incoming path does not flow to the outgoing path during inhalation, CO\(_2\) related
TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/unit</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>[L]</td>
<td>Volume of CO₂ existing in outgoing path</td>
</tr>
<tr>
<td>y</td>
<td>[L]</td>
<td>Volume of CO₂ absorbed by CO₂ scrubber</td>
</tr>
<tr>
<td>p</td>
<td>[L]</td>
<td>Overall CO₂ flow in rebreather</td>
</tr>
<tr>
<td>ν</td>
<td>[L/min]</td>
<td>CO₂ exhalation rate</td>
</tr>
<tr>
<td>c₁</td>
<td>[L/min]</td>
<td>Inhaled rate</td>
</tr>
<tr>
<td>c₂</td>
<td></td>
<td>Ratio of CO₂ in the outgoing path</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>Heart rate ratio (HR/HRₘₐₓ)</td>
</tr>
</tbody>
</table>

scrubber faults do not have influence on CO₂ flow during inhalation. Thus, (5) and (6) still hold.

However, the faults affect CO₂ flow during exhalation. The volume of CO₂ in the outgoing path x is as defined in (15). The volume of CO₂ absorbed by the scrubber y is determined by the difference between the volume of CO₂ arriving at the inlet of a scrubber canister due to exhalation and the volume of a CO₂ channeling event.

During exhalation (t ∈ I₂):

\[
\begin{align*}
\dot{x} &= G(y(t), \mathcal{U}(t,n)) dN(t) \\
\dot{y} &= \mathcal{U}(t,n) dt - dx
\end{align*}
\]

where \( x(0) = 0, \ y(0) = 0 \) and \( I_2 = \left[ \frac{(n+\frac{1}{2})T}{\lambda(t)}, \frac{(n+1)T}{\lambda(t)} \right) \) for \( n = \{0, 1, 2, \cdots \} \).

We define p such that \( p(t) = x(t) + y(t) \) which represents the total CO₂ flow in the system, i.e., how much CO₂ enters or leaves a rebreather. Then, the rebreather system can be modeled as a stochastic system with p(t) and x(t) as state variables.

During inhalation (t ∈ I₁):

\[
\begin{align*}
\dot{x} &= -\frac{v(t)}{V_L} \cdot x(t) dt \\
\dot{p} &= \mathcal{U}(t,n) dt
\end{align*}
\]

During exhalation (t ∈ I₂):

\[
\begin{align*}
\dot{x} &= G(p(t) - x(t), \mathcal{U}(t,n)) dN(t) \\
\dot{p} &= \mathcal{U}(t,n) dt
\end{align*}
\]

where \( x(0) = 0, \ p(0) = 0, \ I_1 = \left[ \frac{nt}{\lambda(t)}, \frac{(n+\frac{1}{2})T}{\lambda(t)} \right) \) and \( I_2 = \left[ \frac{(n+\frac{1}{2})T}{\lambda(t)}, \frac{(n+1)T}{\lambda(t)} \right) \) for \( n = \{0, 1, 2, \cdots \} \).

The volume of CO₂ in the outgoing path x will be measured by a CO₂ sensor. However, the total CO₂ flow p is not able to be measured. Thus, the noisy measurement \( m(t) \) for \( x(t) \) can be modeled as an output of the stochastic systems in (21)-(24).

\[
m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} + v(t)
\]

where \( v(t) \) represents measurement noise.

V. PARTICLE FILTER

Since the model is driven by a Poisson process, there are limited choices of filters for state estimation. In particular, Kalman filter may not be very efficient. In this paper, the particle filter is applied to estimate the states \( x(t) \) and \( p(t) \). We first briefly introduce the particle filtering algorithm [18], and then apply the filter to estimate the states. The particle filter is performed by the iteration of the following four steps.

(i) Particle Creation

To initialize the filter, N samples for each x and p are randomly generated from the initial probability distribution functions \( \rho(x_0) \) and \( \rho(p_0) \). These samples are denoted as \( x_0^i(0) \) and \( p_0^i(0) (i = 1, \cdots, N) \), respectively. The values of the mean and the variance of the initial particles are chosen according to the fact that the volume of CO₂ is always positive and the amount of CO₂ is very small when an event of CO₂ channeling occurs. The values we empirically found are on the order of \( 10^{-4} \). Since these values are very small, A scaling factor \( \gamma \) is used to increase the value of particles to reduce numerical errors. The value used for simulation is \( \gamma = 10^6 \).

(ii) Prediction (Diffusion)

At each time step k, the particles are propagated to the next time step based on the system equations:

During inhalation (k ∈ I₁):

\[
\begin{align*}
x_i^−((k+1)h) &= x_i^−(kh) - \gamma c_1(l)v(kh)h \\
p_i^−((k+1)h) &= p_i^−(kh) + y_i^−((k+1)h) - y_i^−(kh)
\end{align*}
\]

During exhalation (k ∈ I₂):

\[
\begin{align*}
x_i^−((k+1)h) &= x_i^−(kh) + G(kh) dN(t) \\
p_i^−((k+1)h) &= p_i^−(kh) + \mathcal{U}(kh,n)h
\end{align*}
\]

where \( G(kh) = G(p_i^−(kh) - y_i^−(kh), \mathcal{U}(kh,n)), i = \{1, \cdots, N\} \) and \( h \) is the step size.

A Poisson jump \( dN(t) \) is a continuous-time process, but it can be simulated in discrete-time as \( \text{Poiss}(kh) \). Computational implementation of a Poisson process is introduced in [19], [20], [21]. In this paper, a Poisson process is simulated in the way suggested by [19] and \( \text{Poiss}(kh) \) represents the implementation of the Poisson counter during the time interval \( (kh, (k+1)h) \) where \( h \) is a small step size. If a randomly generated number which is uniformly distributed in the interval \([0,1]\) is smaller than \( 1 - e^{-kh} \), then \( \text{Poiss}(kh) = 1 \); otherwise, \( \text{Poiss}(kh) = 0 \). Then, we can express (28) as

\[
x_i^−((k+1)h) = x_i^−(kh) + G(kh)\text{Poiss}(kh)
\]

(iii) Likelihood Evaluation

We can obtain the output values using (25) as

\[
m(kh) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^−(kh) \\ p_i^−(kh) \end{bmatrix} + v(kh)
\]

where \( v(kh) \sim \mathcal{N}(0, R) \).

After measuring \( m(kh) \), the conditional relative likelihood is computed to evaluate \( P(m(kh)|x_i^−(kh), p_i^−(kh)) \). Given that noise \( v \) is Gaussian, for a specific measurement \( m^∗ \), a relative likelihood \( q_i \) can be computed as
\[
q_i = P\left(m_k = m^* \mid x_k = x_i^-(kh), p_k = p_i^-(kh)\right) \\
= P\left(v_k = m^* - h(x_i^-(kh), p_i^-(kh))\right) \\
\sim \exp\left(-\frac{m^* - h(x_i^-(kh), p_i^-(kh))}{2}\right) \\
(2\pi)^{n/2}|R|^{1/2} \\
(32)
\]

where \(r_i(kh) = |m^* - h(x_i^-(kh), p_i^-(kh))|\) and \(n = 1\). Then, in order to ensure that the sum of all the likelihoods is equal to one, the relative likelihoods are normalized as \(\bar{q}_i = q_i / \sum_{j=1}^{N} q_j\).

**Resampling**

Particles are resampled by using the following two steps for \(i = 1, \cdots, N\).

1) Generate a uniformly distributed random number \(r\) on \([0,1]\).

2) If \(\sum_{m=1}^{j-1} \bar{q}_m < r\) but \(\sum_{m=j}^{N} \bar{q}_m \geq r\), then \(\hat{x}^+(kh) = x_j^+(kh)\) and \(\hat{p}^+(kh) = p_j^+(kh)\).

Then, the estimates of \(x\) and \(p\) at time \(kh\) are determined by the statistical mean of the updated particles such that

\[
\begin{align*}
\hat{x}(kh) &= \frac{1}{N} \sum_{i=1}^{N} \{x_i^+(kh)\} \\
\hat{p}(kh) &= \frac{1}{N} \sum_{i=1}^{N} \{p_i^+(kh)\}.
\end{align*}
\]

**VI. FAULT DIAGNOSIS AND PROGNOSIS**

Our CO\textsubscript{2} flow model helps the detection of the CO\textsubscript{2} related scrubber faults. Fault diagnosis and prognosis are achieved using the particle filtering results based on the CO\textsubscript{2} flow model.

**A. Fault Detection and Isolation for Rebreathers**

1) **CO\textsubscript{2} bypass**: The estimate of the volume of CO\textsubscript{2} channeling over one single period of exhalation \(\Delta \tilde{x}\) is obtained using \(\tilde{x}\) such that

\[
\Delta \tilde{x}(n) = \tilde{x}((n+1)T) - \tilde{x}(n) + \frac{1}{2}T),
\]

and the occurrence of CO\textsubscript{2} bypass can be determined by summing the volume of CO\textsubscript{2} channelings over a finite time window such that the decision of CO\textsubscript{2} bypass is made based on the following criterion:

\[
\sum_{i=n-M+1}^{n} \Delta \tilde{x}(i) > H_1
\]

(35)

where \(M\) is the width of the time window, and \(H_1\) is the threshold for detection.

Equation (35) itself is enough to detect CO\textsubscript{2} bypass. However, a more detrimental factor induced by CO\textsubscript{2} bypass fault is the partial pressure of CO\textsubscript{2}, ppCO\textsubscript{2}. We assume that the pressure inside the rebreather is the same as the ambient pressure. Provided that the ambient pressure \(P_{amb}\) and the volume of gas in the outgoing path \(V_L\) are known, ppCO\textsubscript{2} can be computed similarly to ppO\textsubscript{2} in [10] as

\[
ppCO_2 = 100 \cdot c_1 \cdot P_{amb} \cdot \frac{\tilde{x} \cdot P_{amb}}{V_L}
\]

(36)

Then, with a threshold value \(H_2\), we can detect CO\textsubscript{2} bypass by

\[
ppCO_2 > H_2.
\]

(37)

The fault of CO\textsubscript{2} bypass will be detected if either of the two thresholds, \(H_1\) or \(H_2\), are exceeded.

2) **CO\textsubscript{2} scrubber exhaustion and CO\textsubscript{2} scrubber breakthrough**: CO\textsubscript{2} scrubber exhaustion and CO\textsubscript{2} scrubber breakthrough happen when CO\textsubscript{2} absorption capability is lost. To detect these faults we need to compare \(\Delta \tilde{x}\) with the volume of CO\textsubscript{2} exhaled over one breathing period. Let us define

\[
\Delta \mathcal{U}(n) = \int_{(n+\frac{1}{2})T}^{(n+1)T} \mathcal{U}(\tau, n) d\tau
\]

(38)

where \(n = \{0, 1, 2, \cdots \}\).

We can detect scrubber exhaustion or breakthrough by first checking whether

\[
\Delta \mathcal{U}(n) - \Delta \tilde{x}(n) < H_3
\]

(39)

where \(H_3\) is a threshold value. If the threshold is exceeded, then complete CO\textsubscript{2} channeling events is happening. Using \(\tilde{x}\) and \(\hat{p}\) obtained by (33), we can determine whether the fault will lead to scrubber exhaustion or scrubber breakthrough. If the scrubber does not reach its maximum absorption capacity yet, complete CO\textsubscript{2} channelings indicate scrubber breakthrough. Let us define the remaining scrubber capacity (\(\mathcal{C}\)) as

\[
\mathcal{C} = 1 - \frac{y(t)}{y_{max}} = 1 - \frac{p(t) - x(t)}{y_{max}}
\]

(40)

where \(x\) is the volume of CO\textsubscript{2} in the outgoing path, \(y\) is the current CO\textsubscript{2} absorption capacity of the scrubber, \(p\) is the overall CO\textsubscript{2} flow and \(y_{max}\) is the maximum absorption capacity of the scrubber. At the time of the fault occurrence, if

\[
\mathcal{C} < H_4
\]

(41)

where \(H_4\) is a threshold on the scrubber depletion, then the fault is identified as scrubber exhaustion, otherwise scrubber breakthrough.

**B. Prognostics and Health Management**

We can use the state estimates from the Particle Filter to predict future CO\textsubscript{2} channeling occurrences by propagating the state equations of the rebreather CO\textsubscript{2} flow model forward in time. Prognosis for the faults are implemented according to the fault criteria in (35), (37), (39) and (41) using the predicted values \(\hat{x}\) and \(\hat{p}\) obtained by Algorithm 1 instead of estimated values \(\tilde{x}\) and \(\hat{p}\). The fault prediction algorithm at \(t = kh\) for the future states at \(t = (k+1)h\) is as follows. Note that the tilde represents the state estimate and the hat represents the state prediction.
### Algorithm 1  Fault prediction at $t = kh$ for $t = (k+l)h$

1. $\hat{p}_{amb} = p_{amb}$, $\Delta \hat{x} = \Delta x$
2. $[\hat{x}(kh), \hat{p}(kh)] = \text{ParticleFilter}(x(kh), p(kh))$
3. $\hat{x}(kh) = \hat{x}(kh)$, $\hat{p}(kh) = \hat{p}(kh)$
4. for $i = k$ to $i = k+l-1$
5. $\hat{x}(i+1)h = \hat{x}(ih) + d\hat{x}(ih)$
6. $\hat{p}(i+1)h = \hat{p}(ih) + d\hat{p}(ih)$
7. end for
8. /* Check the following fault decision rules using $\hat{x}$ and $\hat{p}$ instead of $\hat{x}$ and $\hat{p}$ in the equations. */
9. if (35) or (37) is met
10. CO$_2$ bypass alarm
11. end if
12. if (39) is met
13. if (41) is met
14. CO$_2$ exhaustion alarm
15. else
16. CO$_2$ breakthrough alarm
17. end if
18. end if

### VII. Simulation Results

We perform simulations of the rebreather based on our stochastic CO$_2$ flow model at a constant depth with ambient pressure as $P_{amb} = 1$ to demonstrate the CO$_2$ related rebreather faults. The parameters we used in simulations are: the number of particles as $N = 800$; the capacity of the scrubber as $y_{max} = 10$; the size of the outgoing path in the breathing loop as $V_t = 30$; the heart rate ratio as $s = 1$; the stretching factor for the function $G$ as $\alpha = 10^{-8}$; the window size for the detection of the CO$_2$ bypass fault as $M = 1$; and the time step as $h = 0.05$.

The CO$_2$ flow patterns under the faults of scrubber exhaustion and scrubber breakthrough are described in Fig. 3. Scrubber exhaustion happens when the scrubber is depleted and Fig. 3(a) shows that there is a dramatic increase in CO$_2$ in the outgoing path after $t = 330$ under the scrubber exhaustion fault. The scrubber breakthrough is intentionally made such that the maximum capacity of the scrubber $y_{max}$ under scrubber breakthrough is reduced to $0.85 \cdot y_{max}$ from the beginning of the simulation. The CO$_2$ flow pattern under scrubber breakthrough is similar to the scrubber exhaustion case, but when scrubber breakthrough occurs, the increase in CO$_2$ in the outgoing path comes earlier as in Fig. 3(b), so there exists more CO$_2$ in the outgoing path in the same amount of dive time after CO$_2$ breakthrough occurs. CO$_2$ bypass happens at the early stages of scrubber exhaustion and scrubber breakthrough as CO$_2$ is getting accumulated in the outgoing path due to CO$_2$ channeling events.

![Fault prediction](image.png)

Fig. 3. The amount of CO$_2$ in the outgoing path under scrubber exhaustion and scrubber breakthrough up to $t = 370$ are shown in (a) and (b), respectively. Their patterns are almost identical except that they happen at different times. Note the difference in values of the state $x$ at time $t$ between the cases of scrubber exhaustion and scrubber breakthrough. CO$_2$ bypass happens due to the accumulated CO$_2$ in the outgoing path before the exhaustion or the breakthrough.

![Parameters for fault diagnosis](image.png)

Fig. 4. Parameters for fault diagnosis are introduced with the threshold values (red lines) in the case of scrubber breakthrough. For CO$_2$ bypass fault detection, $\Delta x$ and ppCO$_2$ are used as in (a) and (b). Parameters for scrubber exhaustion/breakthrough detection are presented in (c) and (d).

To demonstrate the CO$_2$ related fault diagnosis and prognosis algorithms, we implement the particle filter on our stochastic CO$_2$ flow model. An example of fault diagnosis for scrubber breakthrough is presented in Fig. 4. The detection criteria on CO$_2$ bypass fault are shown with threshold values in Figs. 4(a) and 4(b). The occurrence of CO$_2$ bypass will be determined by checking $\Delta \hat{x}$ and estimated ppCO$_2$. The criteria on the scrubber exhaustion/breakthrough are presented in Figs. 4(c) and 4(d). By setting a threshold value for $\Delta U - \Delta \hat{x}$ as 0.04, scrubber exhaustion/breakthrough can be detected, and the remaining capacity of the scrubber can be checked to determine which fault between scrubber exhaustion and scrubber breakthrough happens.

The system propagation for fault prognosis is shown in Fig. 5. We use the same criteria and threshold values as used in previous figures. We see that the particle filter estimates are...
updated before $t = 280$, and system propagation is performed as the red line segments in Fig. 5. Then, a prediction for scrubber exhaustion/breakthrough can be achieved using $\Delta U - \Delta \tilde{x}$ in Fig. 5(a) and the remaining capacity of the scrubber as $100 \cdot \%$ in Fig. 5(b). Compared with the simulation results of fault diagnosis in Figs. 4, our model is optimistic, e.g., the predicted time of the fault occurrence is about 30 seconds later than the actual fault occurrence time. To improve the accuracy of the fault prediction under scrubber breakthrough, the change in the actual maximum capacity of a CO$_2$ scrubber needs to be studied as well.

![Graph](image.png)

Fig. 5. State propagation after $t = 280$ for a scrubber breakthrough prediction. The blue and the red curves indicate the particle filter estimates and the system propagation, respectively. The threshold values are introduced as green lines.

VIII. CONCLUSIONS

We have presented fault diagnosis and prognosis algorithms for three rebreather faults: CO$_2$ bypass, scrubber exhaustion and scrubber breakthrough. As part of our achievements, we have investigated the process of CO$_2$ flow inside a rebreather and the influence of the faults on CO$_2$ flow. Consequently, we have developed a stochastic model of a CO$_2$ channeling event whose cumulative occurrences result in the rebreather faults and a CO$_2$ flow model using the stochastic model. We have shown that the particle filter is applicable to obtain the state estimates of CO$_2$ flow, and the fault diagnosis/prognosis algorithms are tested in simulations based on the particle filter estimates.

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